

PART –A (PHYSICS)

Questions: 1:- 10^{22} If gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of:

- | | |
|--|---|
| (A) $10^8 \frac{\text{N}}{\text{m}^2}$ | (B) $10^3 \frac{\text{N}}{\text{m}^2}$ |
| (C) $10^4 \frac{\text{N}}{\text{m}^2}$ | (D) $10^{16} \frac{\text{N}}{\text{m}^2}$ |

Ans:- Bonus

Pressure is defined as normal force per unit area.

Force is calculated as change in momentum/ time.

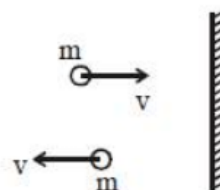
By this answer is 2 N/m^2

None of the option matches so this question must be Bonus.

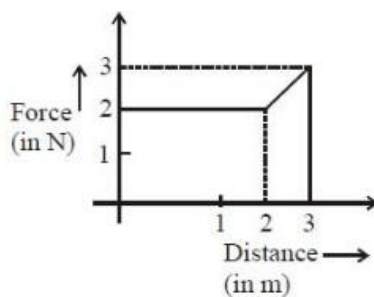
Detailed solution is as following:

Magnitude of change in momentum per collision = $2mv$

$$\begin{aligned} \text{Pressure} &= \frac{\text{Force}}{\text{Area}} = \frac{N(2mv)}{1} \\ &= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1} \\ &= 2 \text{ N/m}^2 \end{aligned}$$



Questions: 2:- A particle moves in one dimension from rest under the influence of a force that varies with the distance traveled by that varies with the distance traveled by the particle as shown in the figure. The kinetic energy of the particle after it has traveled 3 m is:



- | | |
|-----------|-----------|
| (A) 2.5 J | (B) 4 J |
| (C) 5 J | (D) 6.5 J |

Ans:- D

According to work energy theorem.

Work done by force on the particle = Change in KE

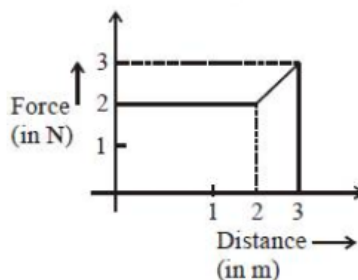
Work done = Area under F-x graph = $\int F \cdot dx =$

$$2 \times 2 + \frac{(2+3) \times 1}{2}$$

$$W = KE_{\text{final}} - KE_{\text{initial}} = 6.5$$

$$KE_{\text{initial}} = 0$$

$$KE_{\text{final}} = 6.5 \text{ J}$$



Questions: 3:- An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be:

- (A) 40 cm from the convergent mirror, same size as the object
- (B) 20 cm from the convergent mirror, same size as the object
- (C) 40 cm from the convergent lens, twice the size of the object
- (D) 20 cm from the convergent mirror, twice the size of the object

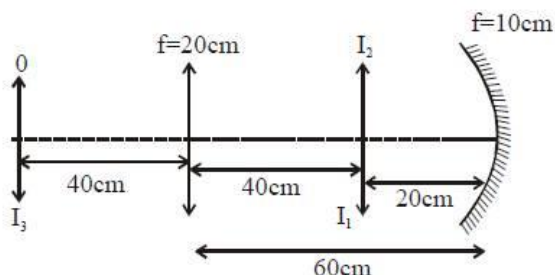
Ans:- Bonus

There will be 3 phenomenon

- (i) Refraction from lens
- (ii) Reflection from mirror
- (iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

None of the option depicts so this question is Bonus.



1st refraction $u = -40 \text{ cm}$; $f = +20 \text{ cm}$

$\Rightarrow v = +40 \text{ cm}$ (image I_1) and $m_1 = -1$

for reflection

$u = -20 \text{ cm}$; $f = -10 \text{ cm}$

$\Rightarrow v = -20 \text{ cm}$ (image I_2) and $m_2 = -1$

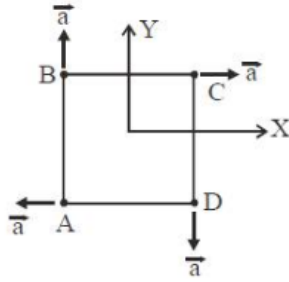
2nd refraction

$u = -40 \text{ cm}$; $f = +20 \text{ cm}$

$\Rightarrow v = +40 \text{ cm}$ (image I_3) and $m_3 = -1$

Total magnification = $m_1 \times m_2 \times m_3 = -1$ and final image is formed at distance 40 cm from convergent lens and is of same size as the object.

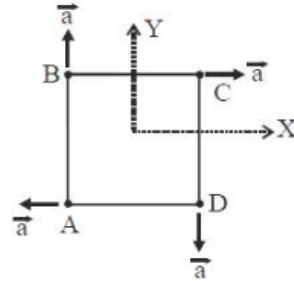
Questions: 4:- Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is:



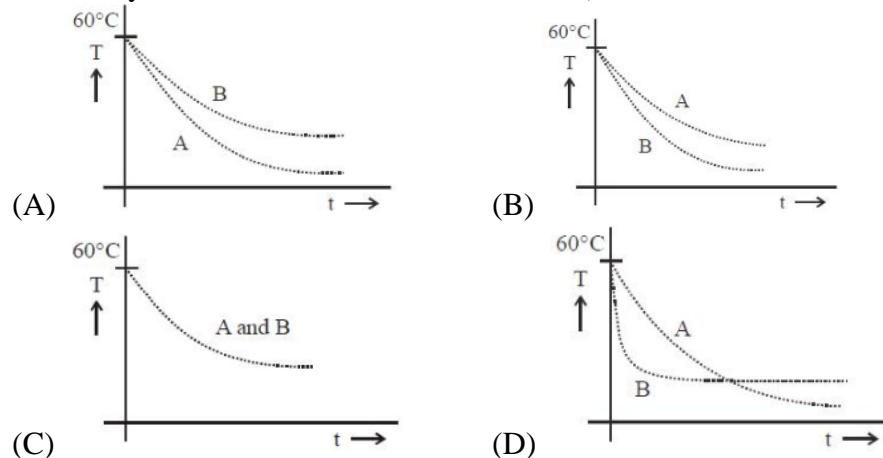
- (A) $\frac{a}{5}(\hat{i} - \hat{j})$ (B) Zero
 (C) $\frac{a}{5}(\hat{i} + \hat{j})$ (D) $a(\hat{i} + \hat{j})$

Ans:- A

$$\begin{aligned}\vec{a}_A &= -a\hat{i} \quad ; \quad \vec{a}_B = a\hat{j} \\ \vec{a}_C &= a\hat{i} \quad ; \quad \vec{a}_D = -a\hat{j} \\ \vec{a}_{cm} &= \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d} \\ \vec{a}_{cm} &= \frac{-ma\hat{i} + 2m\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m} \\ &= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})\end{aligned}$$



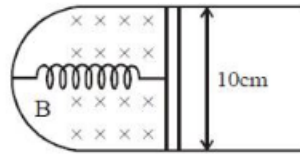
Questions: 5:- Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of $8 \times 10^2 \text{ kg/m}^3$ and specific heat of $2000 \text{ Jkg}^{-1} \text{ K}^{-1}$ while liquid in B has density of 10^3 kgm^{-3} and specific heat of $4000 \text{ JKg}^{-1} \text{ K}^{-1}$. Which of the following best describes their temperature versus time graph schematically? (Assume the emissivity of both the beakers to be the same)



Ans:- A

$$\begin{aligned}
 -ms \frac{dT}{dt} &= e\sigma A(T^4 - T_0^4) \\
 -\frac{dT}{dt} &= \frac{e\sigma A}{ms}(T^4 - T_0^4) \quad ; \quad -\frac{dT}{dt} = \frac{4e\sigma AT_0^3}{ms}(\Delta T) \\
 T &= T_0 + (T_i - T_0)e^{-kt} \\
 \text{where } k &= \frac{4e\sigma AT_0^3}{ms} \\
 k &= \frac{4e\sigma AT_0^3}{\rho v s} \quad ; \quad \left| \frac{dT}{dt} \right| \propto k \\
 \therefore \left| \frac{dT}{dt} \right| &\propto \frac{1}{\rho s} \\
 \rho_A S_A &= 2000 \times 8 \times 10^2 = 16 \times 10^5 \\
 \rho_B S_B &= 4000 \times 10^3 = 4 \times 10^6 \\
 \rho_A S_A &< \rho_B S_B \\
 \left| \frac{dT}{dt} \right|_A &> \left| \frac{dT}{dt} \right|_B
 \end{aligned}$$

Questions: 6:- A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N . If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to:



- (A) 50000
(B) 10000
(C) 1000
(D) 5000

Ans:- D

$$T_0 = 2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{10}}$$

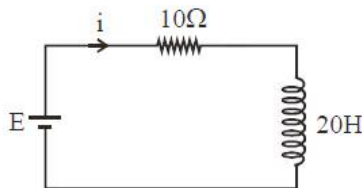
$$A = A_0 e^{-1/\gamma}$$

$$\therefore \text{ for } A = \frac{A_0}{e}, t = \gamma$$

$$t = \gamma = \frac{2m}{b} = \frac{2m}{\frac{B^2 \ell^2}{R}} = 10^4 \text{ s}$$

$$\therefore \text{ No of oscillation } \frac{t}{T_0} = \frac{10^4}{2\pi / \sqrt{10}} \approx 5000.$$

Questions: 7:- A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(A) $\frac{2}{\ln 2}$

(B) $\ln 2$

(C) $\frac{1}{2} \ln 2$

(D) $2 \ln 2$

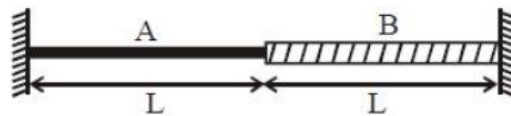
Ans:- D

$$LID = I^2 R$$

$$L \times \frac{E}{10} (-e^{-t/2}) \times \frac{-1}{2} = \frac{E}{10} (1 - e^{-t/2}) \times 10$$

$$e^{-t/2} = 1 - e^{-t/2} \quad ; \quad t = 2 \ln 2$$

Questions: 8:- A wire of length $2L$ is made by joining two wires A and B of same lengths but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio $p : q$ is:



(A) $1 : 4$

(B) $1 : 2$

(C) $3 : 5$

(D) $4 : 9$

Ans:- B

Let mass per unit length of wires are μ_1 and μ_2 respectively.

\therefore Materials are same, so density ρ is same.

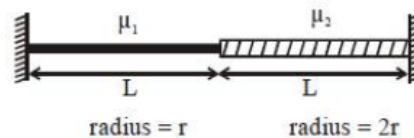
$$\therefore \mu_1 = \frac{\rho \pi r^2 L}{L} = \mu \quad \text{and} \quad \mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension in both are same $= T$, let speed of wave in wires are V_1 and V_2

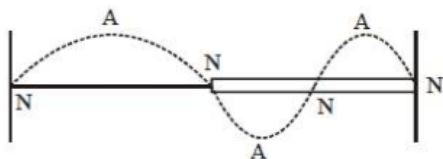
$$V_1 = \frac{V}{2L} = \frac{V}{2L} \quad \& \quad f_{02} = \frac{V_2}{2L} = \frac{V}{4L}$$

Frequency at which both resonate is L.C.M. of

both frequencies i.e. $\frac{V}{2L}$.



Hence number of loops in wires are 1 and 2 respectively



So, ratio of number of antinodes is $1 : 2$.

Questions: 9:- A steel wire having a radius of 2.0 mm , carrying a load of 4 kg , is hanging from a ceiling. Given that $g = 3.1\pi \text{ ms}^{-2}$, what will be the tensile stress that would be developed in the wire?

(A) $6.2 \times 10^6 \text{ Nm}^{-2}$

(B) $4.8 \times 10^6 \text{ Nm}^{-2}$

(C) $5.2 \times 10 \text{ Nm}^{-2}$

(D) $3.1 \times 10^6 \text{ Nm}^{-2}$

Ans:- D

Tensile stress in wire will be

$$= \frac{\text{Tensile force}}{\text{Cross section Area}}$$

$$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} \text{ Nm}^{-2} = 3.1 \times 10^6 \text{ Nm}^{-2}$$

Questions: 10:- Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a $10^6 \frac{\text{V}}{\text{m}}$. maximum electric field of 10 m. The plate area is 10^{-4} m^2 . What is the dielectric constant if the capacitance is 15 pF? (given $\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2/\text{Nm}^2$)

- (A) 3.8 (B) 6.2
(C) 4.5 (D) 8.5

Ans:- D

$$A = 10^{-4} \text{ m}^2$$

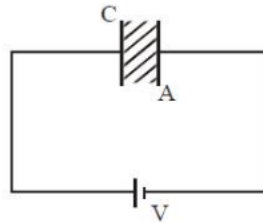
$$E_{\text{max}} = 10^6 \text{ V/m}$$

$$C = 15 \mu\text{F}$$

$$C = \frac{k\epsilon_0 A}{d}; \quad \frac{Cd}{\epsilon_0 A} = k$$

$$k = \frac{15 \times 10^{-12} \times 500 \times 10^{-6}}{8.86 \times 10^{-12} \times 10^{-4}} = \frac{15 \times 5}{8.86} = 8.465$$

$$k \approx 8.5$$



Questions: 11:- An alternating voltage $v(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω . The time taken for the current to rise from half of the peak value of the peak value is:

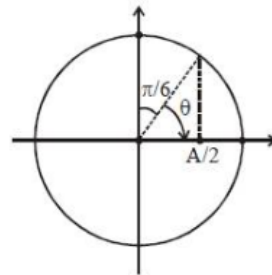
- (A) 2.2 ms (B) 3.3 ms
(C) 5 ms (D) 7.2 ms

Ans:- B

$$V(t) = 220 \sin(100\pi t) \text{ volt time taken,}$$

$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{3}}{100\pi} = \frac{1}{300} \text{ sec}$$

$$= 3.3 \text{ ms}$$



Questions: 12:- The wavelength of the carrier waves in a modern optical fiber communication network is close to:

- (A) 1500 nm (B) 600 nm
(C) 2400 nm (D) 900 nm

Ans:- A

To minimize attenuation, wavelength of carrier waves is close to 1500 nm.

Questions: 13:- Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of: (density of water = 1000kg/m³, coefficient of viscosity of water = 1 mPa s)

- (A) 10³ (B) 10⁶
(C) 10² (D) 10⁴

Ans:- D

$$\text{Reynolds number} = \frac{\rho v d}{\eta}$$

$$\text{Volume flow rate} = v \times \pi r^2$$

$$v = \frac{100 \times 10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}}$$

$$v = \frac{2}{3\pi} \text{ m/s}$$

$$\text{Reynolds number} = \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi} = 2 \times 10^4$$

Order 10⁴

Questions: 14:- A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross – section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20ms⁻¹. Neglect the change in the area of cross section of the cord while stretched. The Young's modulus of rubber is closest to:

- (A) 10³ Nm⁻² (B) 10⁶ Nm⁻²
(C) 10⁸ Nm⁻² (D) 10⁴ Nm⁻²

Ans:- B

$$\text{Energy of catapult} = \frac{1}{2} \times \left(\frac{\Delta \ell}{\ell} \right)^2 \times Y \times A \times \ell$$

$$= \text{Kinetic energy of the ball} = \frac{1}{2} m v^2$$

$$\text{Therefore, } \frac{1}{2} \times \left(\frac{20}{42} \right)^2 \times Y \times \pi \times 3^2 \times 10^{-6} \times 42 \times 10^{-2} = \frac{1}{2} \times 2 \times 10^{-2} \times (20)^2$$

$$Y = 3 \times 10^6 \text{ Nm}^2$$

Questions: 15:- Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ , of the final particle, is given by:

- (A) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (B) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$
(C) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (D) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Ans:- D

$$\odot \rightarrow \frac{h}{\lambda_1} = p_1 \quad \uparrow \quad p_2 = \frac{h}{\lambda_2}$$

$$\vec{p}_1 = \frac{h}{\lambda_1} \hat{i} \quad \text{and} \quad \vec{p}_2 = \frac{h}{\lambda_2} \hat{j}$$

Using momentum conservation

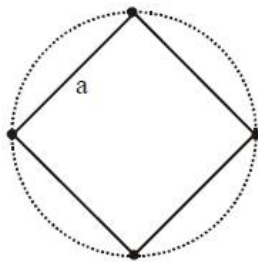
$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \frac{h}{\lambda_1} \hat{i} + \frac{h}{\lambda_2} \hat{j}$$

$$|\vec{p}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

Questions: 16:- Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square?



(A) $1.35\sqrt{\frac{GM}{a}}$

(B) $1.16\sqrt{\frac{GM}{a}}$

(C) $1.41\sqrt{\frac{GM}{a}}$

(D) $1.21\sqrt{\frac{GM}{a}}$

Ans:- B

Net force on particle towards centre of circle is

$$F_c = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2} \sqrt{2}$$

$$= \frac{GM^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right)$$

This force will act as centripetal force. Distance of particle from centre of circle is

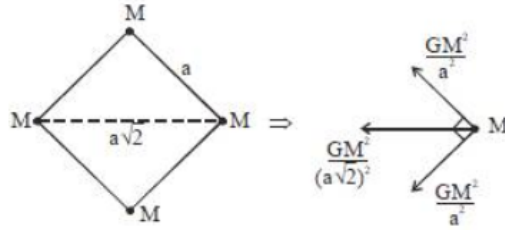
$$\frac{a}{\sqrt{2}}$$

$$r = \frac{a}{\sqrt{2}}, F_c = \frac{mv^2}{r}$$

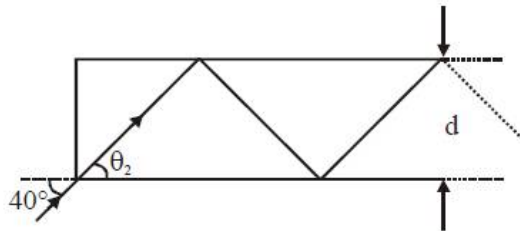
$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right)$$

$$v^2 = \frac{GM}{a} \left(\frac{1}{2\sqrt{2}} + 1 \right)$$

$$v^2 = \frac{GM}{a} (1.35) ; v = 1.16 \sqrt{\frac{GM}{a}}$$



Questions: 17:- In figure, the optical fiber is $l = 2\text{m}$ long and has a diameter of $d = 20\ \mu\text{m}$. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40^\circ$ the number of reflection it makes before emerging from the other end is close to:



- (A) 57000 (B) 45000
(C) 66000 (D) 55000

Ans:- Bonus

If we approximate the angle θ_2 as 30° initially then answer will be closer to 57000. but if we solve thoroughly, answer will be close to 55000.

So both the answers must be awarded. Detailed solution as following.

Exact solution

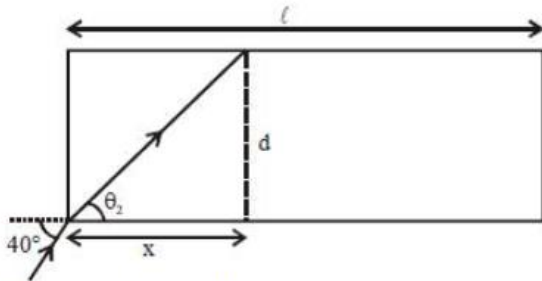
By Snell's law $1 \cdot \sin 40^\circ = (1.31) \sin \theta_2$

$$\sin \theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

$$\text{Now } \tan \theta_2 = \frac{64}{\sqrt{(131)^2 - (64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{d}{x}$$

Now number of reflections

$$= \frac{2 \times 64}{114.3 \times 20 \times 10^{-6}} = \frac{64 \times 10^5}{114.3} \approx 55991 \approx 55000$$

**Approximate solution**

By Snell's law $1 \cdot \sin 40^\circ = (1.31) \sin \theta_2$

$$\sin \theta_2 = \frac{0.64}{1.31} = \frac{64}{131} \approx 0.49$$

If assume $\Rightarrow \theta_2 \approx 30^\circ$

$$\tan 30^\circ = \frac{d}{x} \Rightarrow x = \sqrt{3}d$$

Now number of reflections

$$= \frac{l}{\sqrt{3}d} = \frac{2}{\sqrt{3} \times 20 \times 10^{-6}} = \frac{10^5}{\sqrt{3}} \approx 57735 \approx 57000$$

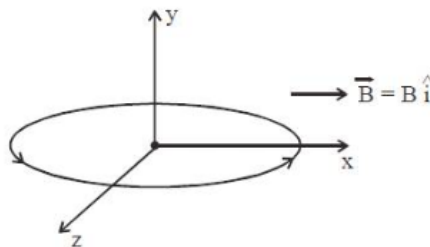
Questions: 18:- A circular coil having N turns and radius r carries a current. It is held in the XZ plane in a magnetic field \hat{i} . The torque on the coil due to the magnetic field is:

- (A) $\frac{Br^2 I}{\pi N}$ (B) zero
(C) $\frac{B\pi r^2 I}{N}$ (D) $B\pi r^2 IN$

Ans:- D

$$\begin{aligned} \text{Magnetic moment of coil} &= NIA \hat{j} \\ &= NI(\pi r^2) \hat{j} \end{aligned}$$

$$\begin{aligned} \text{Torque on loop (coil)} &= \vec{M} \times \vec{B} \\ &= NI(\pi r^2) B \sin 90^\circ (-\hat{k}) \\ &= NI\pi r^2 B (-\hat{k}) \end{aligned}$$



Questions: 19:- Ship A is sailing towards north – east with velocity $\vec{v} = 30\hat{i} + 50\hat{j} \text{ km/hr}$ where \hat{i} points east and \hat{j} , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B ins:

- (A) 2.2 hrs. (B) 4.2 hrs
(C) 2.6 hrs. (D) 3.2 hrs.

Ans:- C

If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as:

$$\vec{v}_A = (30\hat{i} + 50\hat{j}) \text{ km/hr}$$

$$\vec{v}_B = -10\hat{i} \text{ km/hr} ; \quad \vec{r}_A = 0\hat{i} + 0\hat{j}$$

$$\vec{r}_B = (80\hat{i} + 150\hat{j}) \text{ km}$$

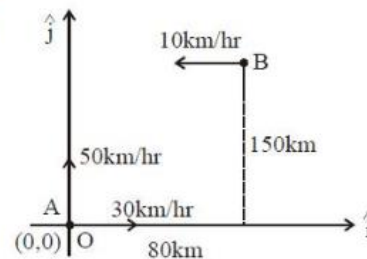
Time after which distance between them will be minimum

$$t = -\frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2};$$

$$\text{Where } \vec{r}_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\vec{v}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j}) = (-40\hat{i} - 50\hat{j}) \text{ km/hr}$$

$$\begin{aligned} \therefore t &= -\frac{(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{j})}{|(-40\hat{i} - 50\hat{j})|^2} \\ &= \frac{3200 + 7500}{4100} \text{ hr} = \frac{10700}{4100} \text{ hr} = 2.6 \text{ hrs} \end{aligned}$$



Questions: 20:- A plane electromagnetic wave travels in free space along the x – direction. The electric field component of the wave at a particular point of space and time is $E = Vm^{-1}$ along y – direction. Its corresponding magnetic field component, B would be:

- (A) $2 \times 10^{-8} \text{ T}$ along z – direction (B) $6 \times 10^{-8} \text{ T}$ along x – direction
(C) $6 \times 10^{-8} \text{ T}$ along z- direction (D) $2 \times 10^{-8} \text{ T}$ along y – direction

Ans:- A

The direction of propagation of an EM wave is direction of $\vec{E} \times \vec{B}$.

$$\hat{i} = \hat{j} \times \hat{B}$$

$$\Rightarrow \hat{B} = \hat{k}$$

$$C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{6}{3 \times 10^8}$$

$$B = 2 \times 10^{-8} \text{ T along z direction.}$$