PART –A (PHYSICS)
JEE MAIN 2018

.***Questions*: 1:-** Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities + σ, –σ and +σ respectively. The potential of shell B is:

(A) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (B) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

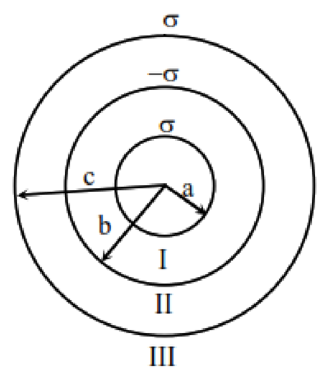
(C) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (D) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

***Ans*:- (C)**

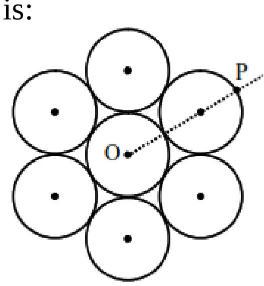
$V_B = V_{B(I)} + V_{B(II)} + V_{B(III)}$

$V_B = \frac{K4\pi a^2\sigma}{b} + \frac{K4\pi b^2(-\sigma)}{b} + \frac{K4\pi c^2(\sigma)}{c}$

$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$



***Questions*: 2:-** Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(A) $\frac{181}{2}MR^2$ (B) $\frac{19}{2}MR^2$

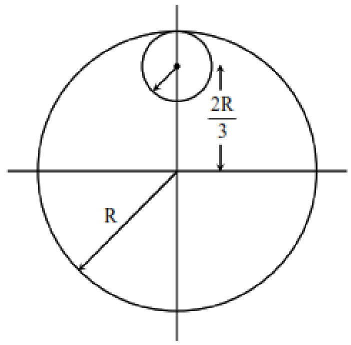
(C) $\frac{55}{2}MR^2$ (D) $\frac{73}{2}MR^2$

***Ans*:- (A)**

$I = I_{cm} + 7M(3R)^2$

$= \left[\frac{MR^2}{2} + 6\times\left\{\frac{MR^2}{2} + M(2R)^2\right\}\right] + 7M(3R)^2 = \frac{181\,MR^2}{2}$

***Questions*: 3:-** From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



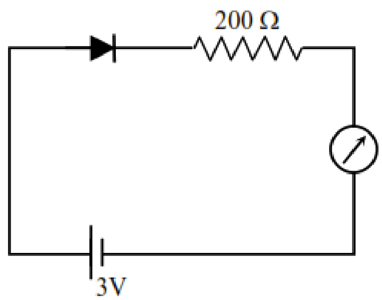
(A) $\frac{37}{9}MR^2$ (B) 4 MR^2

(C) $\frac{40}{9}MR^2$ (D) 10 MR^2

***Ans*:- (B)**

$$I = \frac{9MR^2}{2} - \left[\frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2\right] = 4\ MR^2$$

***Questions*: 4:-** The reading of the ammeter for a silicon diode in the given circuit is:

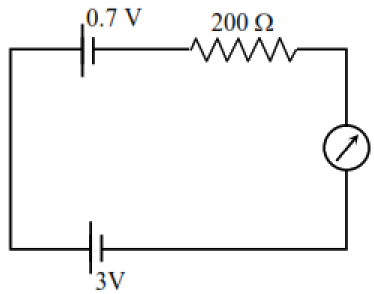


(A) 13.5 mA (B) 0 mA

(C) 15 mA (D) 11.5 mA

***Ans*:- (D)**

$$I = \frac{3-0.7}{200}A = 11.5\ mA$$



***Questions*: 5:-** Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{1}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polarizer A and C is:

(A) 60° (B) 0°

(C) 30° (D) 45°

Ans:- **(D)**

$$\frac{I}{8}=\frac{I}{2}\left(\cos^2\theta\right)^2$$

$$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta=45^\circ$$

Questions: **6:-** For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_o=\frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by:

(A) $\frac{CR}{\omega_0}$ (B) $\frac{\omega_0 L}{R}$

(C) $\frac{\omega_0 R}{L}$ (D) $\frac{R}{(\omega_0 C)}$

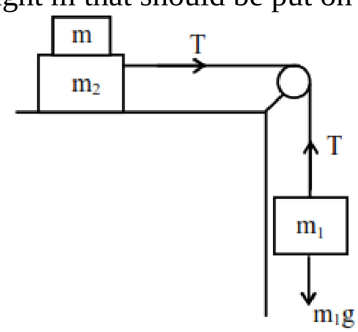
Ans:- **(B)**

$$Q=\frac{\omega_1-\omega_2}{\omega_0}=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{\omega_0 L}{R}\quad [\text{ here } \omega_0^2=\frac{1}{LC}]$$

Alternate solution

$\frac{\omega_0 L}{R}$ is the only dimensionless quantity, hence must be the quality factor.

Questions: **7:-** Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(A) 10.3 kg (B) 18.3 kg

(C) 27.3 kg (D) 43.3 kg

Ans:- **(C)**

$$T=5g$$

$$\mu(10+m)g\geq 5g$$

$$10+m\geq\frac{5}{0\cdot 15}$$

$$m\geq 23.33\text{ kg}$$

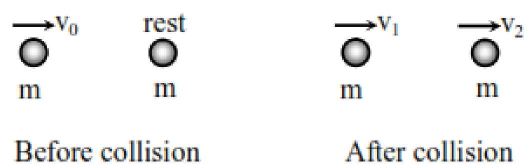
The minimum value from the options, satisfying the above condition is, m = 27.3 kg

Questions: **8:-** In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(A) $\frac{v_0}{\sqrt{2}}$ (B) $\frac{v_0}{\sqrt{4}}$

(C) $\sqrt{2}\, v_0$ (D) $\frac{v_0}{2}$

Ans:- **(C)**



$k_f = 1.5\ k_i$

$v_1^2 + v_2^2 = 1.5 v_0^2$

From conservation of momentum

$v_1 + v_2 = v_0$

from (i) and (ii)

$2v_1v_2 = -0.5\ v_0^2$

So, $v_2 - v_1 = \sqrt{v_2^2 + v_1^2 - 2v_1v_2} = \sqrt{1.5v_0^2 + 0.5v_0^2} = \sqrt{2}\ v_0$

***Questions*: 9:-** A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then:

(A) $T \propto R^{n/2}$ (B) $T \propto R^{3/2}$ for any n

(C) $T \propto R^{\frac{n}{2}+1}$ (D) $T \propto R^{(n+1)/2}$

Ans:- **(D)**

$F \propto \frac{1}{R^n}$

$v \propto \frac{1}{R^{\frac{n-1}{2}}}$

$T = \frac{2\pi R}{v}$

$T \propto R^{1+\frac{n-1}{2}}$

$T \propto R^{\frac{n+1}{2}}$

***Questions*: 10:-** Two batteries with e. m. f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between:

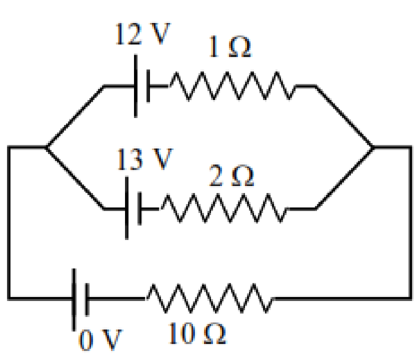
(A) 11.7 V and 11.8 V (B) 11.6 V and 11.7 V

(C) 11.5 V and 11.6 V (D) 11.4 V and 11.5 V

Ans:- **(C)**

The circuit may be drawn as shown in the figure.

$\varepsilon_{eq} = \frac{\Sigma \frac{\varepsilon_i}{r_i}}{\Sigma \frac{1}{r_i}} = 11.56\ \text{V}$



***Questions*: 11:-** In an a. c. circuit, the instantaneous e. m. f. and current are given by

$e = 100 \sin 30\,t$

$i = 20 \sin\left(30\,t - \frac{\pi}{4}\right)$

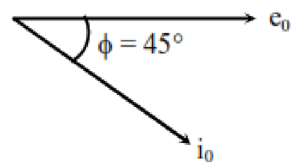
In one cycle of a. c., the average power consumed by the circuit and the wattless current are, respectively:

(A) 50, 0
(B) 50, 10
(C) $\frac{1000}{\sqrt{2}}, 10$
(D) $\frac{50}{\sqrt{2}}, 0$

***Ans*:- (C)**

$$\langle P \rangle = \frac{\varepsilon_0 I_0 \cos\phi}{2} = \frac{(100)(20)\cos\frac{\pi}{4}}{2} = \frac{1000}{\sqrt{2}}\text{ watt}$$

$$\text{Wattless current} = I_{rms}\sin\phi = \frac{20}{\sqrt{2}}\frac{1}{\sqrt{2}} = 10\text{A}$$



***Questions*: 12:-** An EM wave from air enters a medium. The electric fields are

$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z - ct)\right]$

in medium, where the wave number k and frequency *ν* refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is **correct**?

(A) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$
(B) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$
(C) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
(D) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

***Ans*:- (D)**

$$\frac{v_{air}}{v_{med}} = \frac{c}{c/2} = 2 = \frac{\sqrt{\mu_0 \epsilon_0 \mu_{r_2} \epsilon_{r_2}}}{\sqrt{\mu_0 \epsilon_0 \mu_{r_1} \epsilon_{r_1}}}$$

$$\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

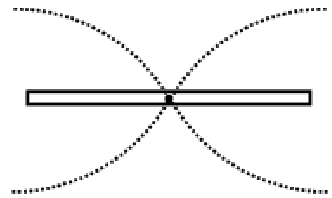
***Questions*: 13:-** A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(A) 2×10^6
(B) 2×10^3
(C) 2×10^4
(D) 2×10^5

***Ans*:- (D)** No. of telephonic channels that can be transmitted simultaneously = $\frac{0.1 \times 10 \times 10^9}{5 \times 10^3} = 2 \times 10^5$

***Questions*: 14:-** A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m and its Young's modulus is 9.27 $\times 10^{10}$ Pa. What will be the fundamental frequency of the longitudinal vibrations?

(A) 7.5 kHz (B) 5 kHz
(C) 2.5 kHz (D) 10 kHz
Ans:- **(B)**

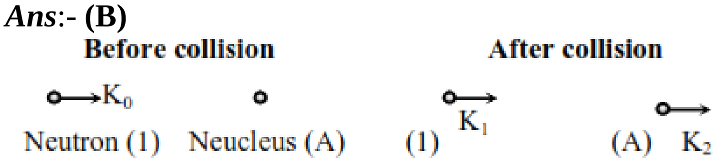


Wave velocity (v) = $\sqrt{\frac{Y}{\rho}} = 5.86\times10^{3}$ m/s

For fundamental mode, $\lambda = 1.2$ m

$\therefore$ fundamental frequency = $\frac{v}{\lambda} = 4.88$ kHz ≈ 5 kHz

***Questions*: 15:-** It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively:
(A) (0, 1) (B) (·89, ·28)
(C) (·28, ·89) (D) (0, 0)
Ans:- **(B)**



$\therefore \sqrt{K_0} = \sqrt{K_1} + \sqrt{AK_2}$ (from conservation of momentum)
and $K_0 = K_1 + K_2$ (for elastic collision)
So after solving

$$(1+A)\frac{K_1}{K_0} - 2\sqrt{\frac{K_1}{K_0}} = (A-1)$$

For Deuterium, A = 2, $1-\frac{K_1}{K_0} = 0.89$

For Carbon, A = 12, $1-\frac{K_1}{K_0} = 0.28$

***Questions*: 16:-** The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:
(A) 6% (B) 2.5%
(C) 3.5% (D) 4.5%
Ans:- **(D)**

$$\frac{\Delta\rho}{\rho}\times100 = \frac{\Delta m}{m}\times100 + \frac{3\times\Delta a}{a}\times100$$
$= 4.5\%$

***Questions*: 17:-** Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(A) (a) 195 K (b) 2.7 kJ
(B) (a) 189 K (b) 2.7 kJ
(C) (a) 195 K (b) –2.7 kJ
(D) (a) 189 K (b) – 2.7 kJ
Ans:- **(D)**
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$\Rightarrow \quad T_2 = 189\,\text{K}$
$\Delta U = nC_v\Delta T = -2.7\,\text{kJ}$

***Questions*: 18:-** A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is:
(A) $\frac{mg}{Ka}$ (B) $\frac{Ka}{mg}$
(C) $\frac{Ka}{3\,mg}$ (D) $\frac{mg}{3\,Ka}$
Ans:- **(D)**
$\frac{\Delta V}{V} = \frac{3\Delta r}{r}$
$K = \frac{P}{\Delta V / V}$
$\Rightarrow \quad \frac{\Delta r}{r} = \frac{mg}{3Ka}$

***Questions*: 19:-** A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:
(A) 0.9 n C (B) 1.2 n C
(C) 0.3 n C (D) 2.4 n C
Ans:- **(B)** Q KCV
$Q_{induced} = Q\,(1 - 1/K)$
$\Rightarrow Q_{induced} = 1.2$ n C

***Questions*: 20:-** The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:
(A) $\frac{1}{\sqrt{2}}$ (B) 2
(C) $\sqrt{3}$ (D) $\sqrt{2}$
Ans:- **(D)**

$$m = I \times \pi r^2$$
$$2m = I \times \pi (r')^2$$
$$\Rightarrow \quad r' = \sqrt{2} r$$
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow \frac{B_1}{B_2} = \frac{r'}{r} = \sqrt{2}$$