Chapter 4 Waves

Day - 1

INTRODUCTION

Ripples on a pond, musical sounds, seismic tremors triggered by an earthquake- all these are wave phenomena. Waves can occur whenever a system is disturbed from equilibrium and when the disturbance can travel, propagate, from one region of the system to another. As a wave propagates, it carries energy. The energy in light waves from the sun warms the surface of our planet; the energy in seismic waves can crack our planet's crust.

This chapter and the next are about mechanical waves-waves that travel within some material called medium. We'll begin this chapter by deriving the basic equations for describing waves, including the important special case of sinusoidal waves in which the wave pattern is a repeating sine or cosine function. To help us understand waves in general, we'll look at the simple case of waves that travel on a stretched string or rope.

Waves on a string play an important role in music. When a musician strums a guitar or bows a violin, she makes waves that travel in opposite directions along the instrument's strings. What happens when these oppositely directed waves overlap is called interference. We'll discover that sinusoidal waves can occur on a guitar or violin string only for certain special frequencies, called normal-mode frequencies, determined by the properties of the string. The normal-mode frequencies of a stringed instrument determine the pitch of the musical sounds that the instrument produces. (In the next chapter we'll find that interference also helps explain the pitches of wind instruments such as flutes and pipe organs.)

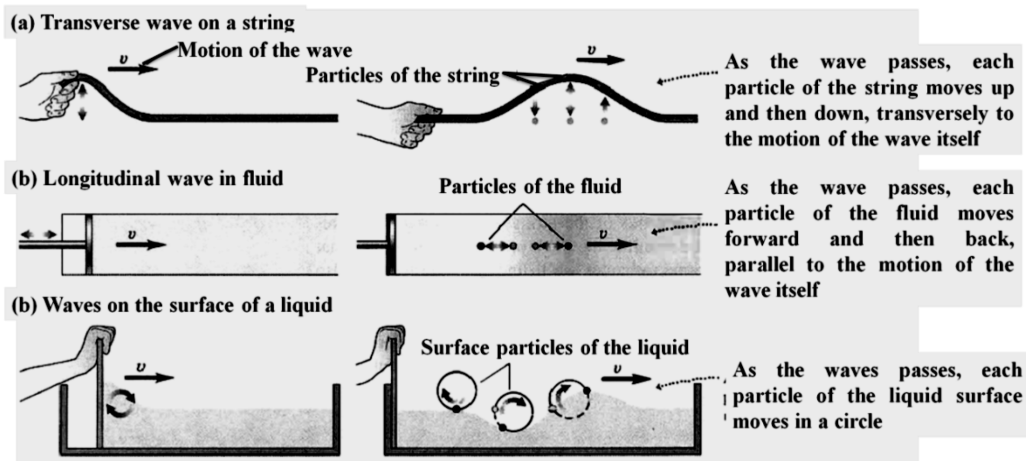
Not all waves are mechanical in nature. Electromagnetic waves-including light, radio waves, infrared and ultraviolet radiation, and x rays-can propagate even in empty space, where there is no medium. We'll explore these and other non mechanical waves in later chapters.

TYPES OF MECHANICAL WAVES

A mechanical wave is a disturbance that travels through some material or substance called the medium for the wave. As the wave travels through the medium, the particles that make up the medium undergo displacements of various kinds, depending on the nature of the wave. Shows three varieties of mechanical waves. The medium is a string or rope under tension. If we give the left end a small upward shake or wiggle, the wiggle travels along the length of the string. Successive sections of string go through the same motion that we gave to the end, but at successively later times. Because the displacements of the medium are perpendicular or transverse to the direction of travel of the wave along the medium, this is called a **transverse wave.**

The medium is a liquid or gas in a tube with a rigid wall at the right end and a movable piston a the left end. If we give the piston a single back-and-forth motion, displacement and pressure fluctuations travel down the length of the medium. This time the motions of the particles of the medium are back and forth along the same direction that the wave travels. We call this a **longitudinal wave.**

The medium is a liquid in a channel, such as water in an irrigation ditch or canal. When we move the flat board at the left end forward and back once, a wave disturbance travels down the length of the channel. In this case the displacements of the water have both longitudinal and transverse components.



Each of these systems has an equilibrium state. For the stretched string it is the state in which the system is at rest, stretched out along a straight line. For the fluid in a tube it is a state in which the fluid is at rest with uniform pressure. And for the liquid in a trough it is a smooth, level water surface. In each case the wave motion is a disturbance from the equilibrium state that travels from one region of the medium to another. And in each case there are forces that tend to restore the system to its equilibrium position when it is displaced, just as the force of gravity tends to pull a pendulum toward its straight-down equilibrium position when it is displaced.

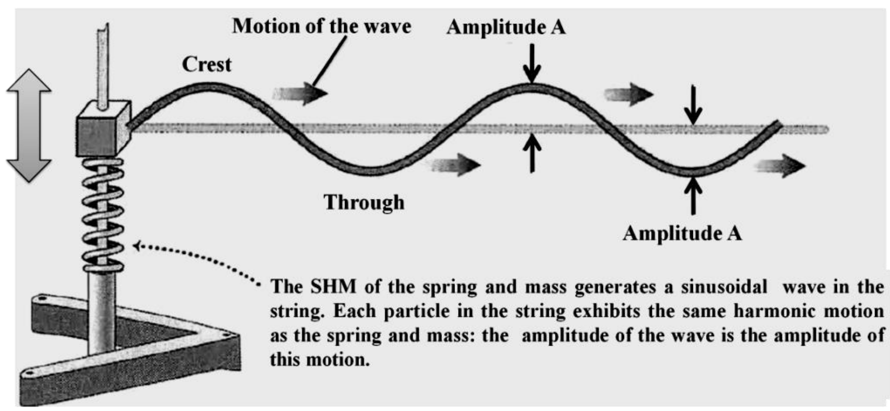
These examples have three things in common. First, in each case the disturbance travels or propagates with a definite speed through the medium. This speed is called the speed of propagation, or simply the wave speed. Its value is determined in each case by the mechanical properties of the medium. We will use the symbol v for wave speed. (The wave speed is not the same as the speed with which particles move when they are disturbed by the wave. Second, the medium itself does not travel through space; its individual particles undergo back-and-forth or up-and-down motions around their equilibrium positions. The overall pattern of the wave disturbance is what travels. Third, to set any of these systems into motion, we have to put in energy by doing mechanical work on the system. The wave motion transports this energy from one region of the medium to another.

PERIODIC WAVES

The transverse wave on a stretched string in is an example of a wave pulse. The hand shakes the string upand down just once, exerting a transverse force on it as it does so. The result is a single "wiggle," or pulse, that travels along the length of the string. The tension in the string restores its straight-line shape once the pulse has passed.

A more interesting situation develops when we give the free end of the string a repetitive, or periodic, motion. Then each particle in the string also undergoes periodic motion as the wave propagates, and we have a periodic wave.

In particular, suppose we move the string up and down with simple harmonic motion (SHM) with amplitude A, frequency 1, angular frequency $\omega = 2\pi f$, and period $T = 1/f = 2\pi/\omega$. Figure shows one way to do this. The wave that results is a symmetrical sequence of crests and troughs. As we will see, periodic waves with simple harmonic motion are particularly easy to analyze; we call them **sinusoidal waves.** It also turns out that any periodic wave can be represented as a combination of sinusoidal waves. So this particular kind of wave motion is worth special attention.



The SHM of the spring and mass generates a sinusoidal wave in the string. Each particle in the string exhibits the same harmonic motion as the spring and mass: the amplitude of the wave is the amplitude of this motion.

The wave that advances along the string is a continuous succession of transverse sinusoidal disturbances. Figure shows the shape of a part of the string near the left end at time intervals of $1/8$ of a period, for a total time of one period. The wave shape advances steadily toward the right, as indicated by the highlighted area. As the wave moves, any point on the string (any of the red dots, for example) oscillates up and down about its equilibrium position with simple harmonic motion. When a sinusoidal wave passes through a medium, every particle in the medium undergoes simple harmonic motion with the same frequency.

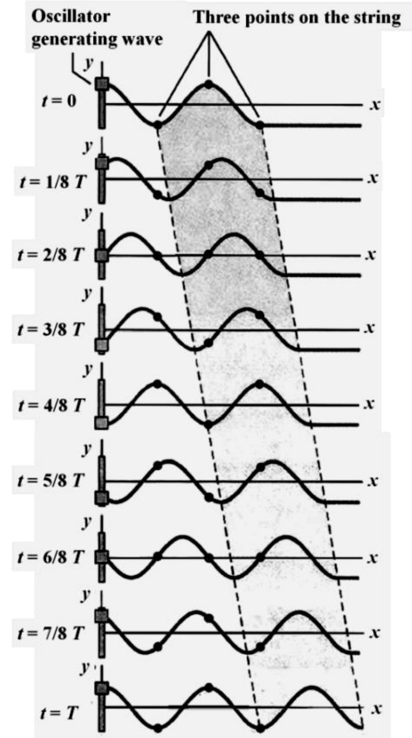
Wave motion vs. particle motion be very careful to distinguish between the motion of the transverse wave along the string and the motion of a particle of the string. The wave moves with constant speed v along the length of the string, while the motion of the particle is simple harmonic and transverse (perpendicular) to the length of the string.

For a periodic wave, the shape of the string at any Instant is a repeating pattern. The length of one complete Wave pattern is the distance from one crest to the next, or From one trough to the next, or from any point to the

Corresponding point on the next repetition of the wave shape. We call this distance the wavelength of the wave, denoted by λ (the Greek letter lambda).

The wave pattern travels with constant speed u and advances a distance of one wavelength λ in a time interval of one period T. So the wave speed v is Given by $v = \lambda/T$, because $f = 1/T$.

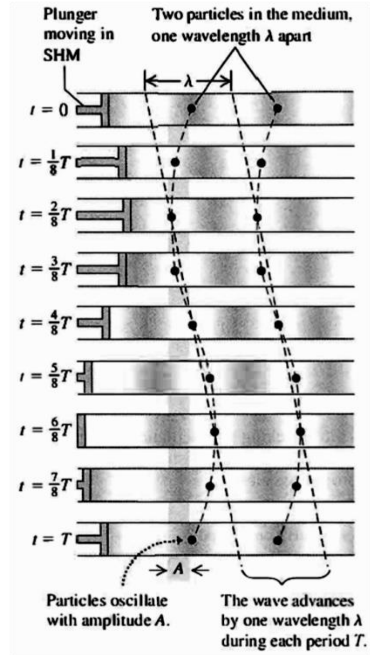
$$v = \lambda f \qquad \text{(periodic wave)}$$



The speed of propagation equals the product of wavelength and frequency. The frequency is a property of the entire periodic wave because all points on the string oscillate with the same frequency of waves on a String propagate in just one dimension. But the ideas of frequency, wavelength, and amplitude apply equally well to waves that propagate in two or three dimensions. Shows a wave propagating in two dimensions on the surface of a tank of water. As with waves on a string, the wavelength is the distance from one crest to the next, and the amplitude is the height of a crest above the equilibrium level. In many important situations including waves on a string, wave speed v is determined entirely by the mechanical properties of the medium. In this case, increasing f causes λ to decrease so that the product $v = \lambda f$ remains the same, and waves of all frequencies propagate with the same wave speed. In this chapter we will consider only waves of this kind. (In later chapters we will study the propagation of light waves in matter for which the wave speed depends on frequency; this turns out to be the reason prisms break white light into a spectrum and raindrops create a rainbow.)

To understand the mechanics of a periodic longitudinal wave, we consider a long tube filled with a fluid, with a piston at the left end as in Fig. If we push the piston in, we compress the fluid near the piston, increasing the pressure in this region. This region then pushes against the neighboring region of fluid, and so on, and a wave pulse moves along the tube. Now suppose we move the piston back and forth with simple harmonic motion, along a line parallel to the axis of the tube. This motion forms regions in the fluid where the pressure and density are greater or less than the equilibrium values. We call a region of increased density a compression; a region of reduced density is a rarefaction. Shows compressions as darkly shaded areas and rarefactions as lightly shaded areas. The wavelength is the distance from one compression to the next or from one rarefaction to the next.

The wave propagating in the fluid-filled tube at time intervals of 1/8 of a period, for a total time of one period. The pattern of compressions and rarefactions moves steadily to the right, just like the pattern of crests and troughs in a sinusoidal transverse wave. Each particle in the fluid oscillates in SHM parallel to the direction of wave propagation (that is, left and right) with the same amplitude A and period T as the piston. The particles shown by the two red dots are one wavelength apart, and so oscillate in phase with each other.



Just like the sinusoidal transverse wave, in one period T the longitudinal wave travels one wavelength λ to the right. Hence the fundamental equation $v = \lambda f$ holds for longitudinal waves as well as for transverse waves, and indeed for all types of periodic waves. Just as for transverse waves, in this chapter and the next we will consider only situations in which the speed of longitudinal waves does not depend on the frequency.

WAVE FUNCTION FOR A SINUSOIDAL WAVE

Let's see how to determine the form of the wave function for a sinusoidal wave. Suppose a sinusoidal wave travels from left to right (the direction of increasing x) along the string. Every particle of the string oscillates with simple harmonic motion with the same amplitude and frequency. But the oscillations of particles at different points on the string are not all in step with each other. The particle at point B is at its maximum positive value of y at $t = 0$ and returns to $y = 0$ at t = 2/8 T; these same events occur for a particle at point A or point C at $t = 4/8\ T$ and $t = 6/8T$, exactly one half-period later. For any two particles of the string, the motion of the particle on the right (in terms of the wave, the "downstream" particle) lags behind the motion of the particle on the left by an amount proportional to the distance between the particles.

Hence the cyclic motions of various points on the string are out of step with each other by various fractions of a cycle. We call these differences phase differences, and we say that the phase of the motion is different for different points. For example, if one point has its maximum positive displacement at the same time that another has its maximum negative displacement, the two are a half cycle out of phase. (This is the case for points A and B, or points B and C.)

Suppose that the displacement of a particle at the left end of the string ($x = 0$), where the wave originates, is given by –

$$y(x = 0, t) = A \sin \omega t = A \sin 2\pi f t.$$

That is, the particle oscillates in simple harmonic motion with amplitude A, frequency f, and angular frequency $\omega = 2\pi f$. The notation y (x = 0, t) reminds us that the motion of this particle is a special case of the wave function y (x, t) that describes the entire wave. At $t = 0$ the particle at x = 0 is at its maximum positive displacement (y = A) and is instantaneously at rest (because the value of y is a maximum).

The wave disturbance travels from x = 0 to some point x to the right of the origin in an amount of time given by x/v, where v is the wave speed. So the motion of point x at time t is the same as the motion of point x = 0 at the earlier time $t - x/v$. Hence we can find the displacement of point x at time t by simply replacing t by $(t - xlv)$. When we do that, we find the following expression for the wave function:

$$y(x,t) = A\sin\omega\left(t - \frac{x}{v}\right) = a\sin 2\pi f\left(t - \frac{x}{v}\right) \quad \text{(Sinusoidal wave moving in + } x\text{-direction)}$$

The displacement $y(x, t)$ is a function of both the location x of the point and the time t. We could make more general by allowing for different values of the phase angle, as we did for simple harmonic motion but for now we omit this. We can rewrite the wave function given by several different but useful forms. We can express it in terms of the period T = $1/f$ and the wavelength $\lambda = v/f$:

$$y(x,t) = A\sin 2\pi\left(\frac{1}{T} - \frac{x}{v}\right) \quad \text{(Sinusoidal wave moving in + } x\text{-direction)}$$

We get another convenient form of the wave function if we define a quantity k, called the wave number:

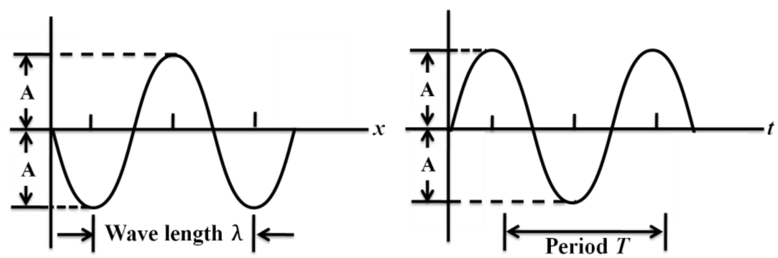
$$k = \frac{2\pi}{\lambda} \quad \text{(wave number)}$$

Substituting $\lambda = 2\pi/k$ and $f = \omega/2\pi$ into the wavelength-frequency relationship $v = \lambda f$ gives

$$\omega = vk \quad \text{(periodic wave)}$$

$$y(x,t) = A\sin(\omega t - kx) \quad (\text{Sinusoidal wave moving in} + x - \text{direction})$$

Which of these various forms for the wave function y (x, t) we use in any specific problem is a matter of convenience. Note that ω has units rad/s, so for unit consistency the wave number k must have the units rad/m. (Some physicists define the wave number as $1/\lambda$ rather than $2\pi/\lambda$. When reading other texts, be sure to determine how this term is defined.)



The wave function y (x, t) is graphed as a function of x for a specific time t. This graph gives the displacement y of a particle from its equilibrium position as a function of the coordinate x of the particle. If the wave is a transverse wave on a string, the graph represents the shape of the string at that instant, like a flash photograph of the string. In particular, at time t = 0,

$$y(x, t = 0) = A\sin(-kx) = -A\sin kx = -A\sin 2\pi\frac{x}{\lambda}.$$

A graph of the wave function versus time t for a specific coordinate x. This graph gives the displacement y of the particle at that coordinate as a function of time; that is, it describes the motion of that particle. In particular, at the position x = 0,

$$y(x, = 0, t) = A\sin\omega t = A\sin 2\pi\frac{t}{T}.$$

$$y(x,t) = A\sin 2\pi f\left(t + \frac{x}{v}\right) = A\sin 2\pi\left(\frac{1}{T} + \frac{x}{\lambda}\right)$$

$$= A\sin(\omega t + kx)\ \ \text{(sinusoidal wave moving in} - x - \text{direction)}$$

In the expression $y(x,t) = A\sin(\omega t \pm kx)$ for a wave traveling in the –*x* or + *x*-direction, the quantity $(\omega t \pm kx)$ is called the phase. It plays the role of an angular quantity (always measured in radians) and its value for any values of x and t determines what part of the sinusoidal cycle is occurring at a particular point and time. For a crest (where *y* = A and the cosine function has the value 1), the phase could be $\pi/2, 5\pi/2,$ and so on; for a trough (where *y* = –A and the cosine has the value –1), it could be $\pi, 3\pi, 5\pi,$ and so on.

The wave speed is the speed with which we have to move along with the wave to keep alongside a point of a given phase, such as a particular crest of a wave on a string. For a wave traveling in the + *x* direction, that means $\omega t - kx$ = constant. Taking the derivative with respect to *t*, we find $\omega = kdx/dt$, or

$$\frac{dx}{dt} = \frac{\omega}{k}.$$

Comparing this we see that *dx/dt* is equal to the speed *v* of the wave. Because of this relationship, *v* is sometimes called the phase velocity of the wave. (Phase speed would be a better term.)

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