## Day - 1

## VISCOSITY

According to Newton, the frictional force F (or viscous force) between two layers depends upon the following factors,
(i) Force F is directly proportional to the area (A) of the layers in contact, i.e. ,
$\mathrm{F} \propto \mathrm{A}$
(ii) Force F is directly proportional to the velocity gradient $\frac{d v}{d t}$ between the layers, combining these two, we have
$F \propto A \frac{d v}{d y}$
$F=-\eta A \frac{d v}{d y}$
Here, $\mathfrak{\eta}$ is constant of proportionality and is called coefficient of viscosity .its value depends on the nature of the fluid. The negative sign in the above equation shows that the direction of viscous force F is opposite to the direction of relative velocity of the layer.
The S.I unit of y is $\mathrm{N} \mathrm{s} / \mathrm{m}^{2}$
It is also called decapoise or Pascal second. Thus,
1 decapoise $=1 N \frac{s}{m^{2}}=1 \mathrm{~Pa} \times s=10$ poise
Dimensions of $\eta$ are $\left[\mathrm{ML}^{-1} \mathrm{~T}^{-1}\right]$
Coefficient of viscosity of water at $10^{\circ} \mathrm{C}$ in $\mathrm{y}=1.3 \times 10^{-3} \mathrm{~N} \mathrm{~s} / \mathrm{m}^{2}$
Experiments show that coefficient of viscosity of a liquid decreases as its temperature rises.

## FLOW OF LIQUID THROUGH A CYLINDRICAL PIPE

$v=\frac{P_{1}-P_{2}}{4 \eta L}\left(R^{2}-r^{2}\right)$
Where $P_{1}$ and $P_{2}$ are the pressure at two ends of a pipe with length $L$. The flow is always in the direction of decreasing pressure.
From the above equation we can see that v-r graph is a parabola.
$\mathrm{v}=0 \quad$ at $\quad \mathrm{r}=\mathrm{R} \quad$ (along the wall)
And $v=\frac{\left(P_{1}-P_{2}\right) R^{2}}{4 \eta L}=v_{\text {max }}$ at $\mathrm{r}=0$ (along the axis)
Volume flow rate ( Q or $\frac{d v}{d t}$ )
$Q=\frac{d v}{d t}=\frac{\pi}{8}\left(\frac{R^{4}}{\eta}\right)\left(\frac{P_{1}-P_{2}}{L}\right)$

## STOKES LAW AND TERMINAL VELOCITY

$F=6 \pi \eta r v . \quad(\eta=$ coefficient of viscosity $)$
This law is called stokes law.

## TERMINAL VELOCITY ( $v_{T}$ )

Consider a small sphere falling from rest through a large column of viscous fluid. The forces acting on the sphere are
(i) weight W of the sphere acting vertically downwards
(ii) Upthrust $\mathrm{F}_{1}$ acting vertically upwards.
(iii) viscous force $\mathrm{F}_{\mathrm{v}}$ acting vertically upwards, i.e, in a direction opposite to velocity of the sphere.
Initially,
$\mathrm{F}_{\mathrm{v}}=0$
$\mathrm{W}>\mathrm{F}_{\mathrm{t}}$
and the sphere acceleration downwards. As the velocity of the sphere increases, $\mathrm{F}_{\mathrm{v}}$ increases.
Eventually a stage in reached when
$W=F_{t}+F_{v}$
After this net force of the sphere is zero and it moves downwards with a constant velocity called terminal velocity $\left(\mathrm{v}_{\mathrm{T}}\right)$.
Substituting proper values in equation (i) we have,
$\frac{4}{3} \pi r^{3} \rho g=\frac{4}{3} \pi r^{3} \sigma g+6 \pi \eta r v_{r}$
Here,
$\rho=$ density of sphere
$\sigma=$ density of fluid
And $\eta=$ coefficient of viscosity of fluid
From equation (i) and (ii), we have get

$v_{r}=\frac{2}{9} r^{2} \frac{(\rho-\sigma) g}{\eta}$

## Case

Combination of liquid flow in tubes
Volume of liquid flow per unit time
$V=\frac{\pi P r^{4}}{8 \eta l}$
Liquid resistance

$R=\frac{P}{V}=\frac{8 \eta l}{\pi r^{4}}$

## SERIES COMBINATION

Here Pressure difference $\mathrm{P}=\mathrm{P}_{1}+\mathrm{P}_{2}$
$V_{1}=\frac{\pi P_{1} r_{1}^{4}}{8 \eta l_{1}}, \quad V_{2}=\frac{\pi P_{2} r_{2}^{4}}{8 \eta l_{2}}$

$V_{1}=V_{2}$
$P=P_{1}+P_{2}$
$R_{e q}=R_{1}+R_{2}$

## PARALLEL COMBINATION

Here
$P=P_{1}=P_{2}$
$V=V_{1}+V_{2}$
$=\frac{P \pi r_{1}^{4}}{8 \eta l_{1}}+\frac{p \pi r_{2}^{4}}{8 \eta l_{2}}$
$\frac{1}{R_{e q}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$


Example- A boat of area $10 \mathrm{~m}^{2}$ floating on the surface of a river is made to move horizontally with a speed of $2 \mathrm{~m} / \mathrm{s}$ by applying a tangential force. If the river is 1 m deep and water in contact with the bed is stationary, find the tangential force needed to keep the boat moving with same velocity. Viscosity of water is 0.01 poise.
(a) 0.02 N
(b) 0.01 N
(c) 0.04 N
(d) 0.03 N

Solution: A velocity changes from $2 \mathrm{~m} / \mathrm{s}$ at the surface to zero at the bed which is at depth of 1 m ,
Velocity gradient $=\frac{d v}{d y}=\frac{2-0}{1}=2 s^{-1}$
Now from Newton's law of viscous force,
$|F|=\eta A \frac{d v}{d y}$
$=\left(10^{-2} \times 10^{-1}\right) \times 10 \times 2$
$=0.02 \mathrm{~N}$
Answer (B)
So to keep the boat moving at same velocity, force equal to viscous force, i.e., 0.02 N must be applied.
Example-Find the viscosity of glycerin (having density $1.3 \mathrm{~g} / \mathrm{cc}$ ) if a steel ball of 2 mm radius (density $=8 \mathrm{~g} / \mathrm{cc}$ ) acquires a terminal velocity of $4 \mathrm{~cm} / \mathrm{s}$ in falling freely in the tank of glycerin.
(a) 14.6 poise
(b) 1.46 poise
(c) 146 poise
(d) 56 poise

Solution : We know that
$v_{T}=\frac{2}{9} \frac{g(\rho-\sigma) r^{2}}{\eta}$,
i.e., $\quad \eta=\frac{2}{9} g(\rho-\sigma) \frac{r^{2}}{v_{T}}$

So, $\quad \eta=\frac{2}{9} \times \frac{980 \times(8-1.3) \times(0.2)^{2}}{4}$
$\simeq 14.6$ poise
Answer (A)

Example- An air bubble of radius 1 mm is allowed to rise through a long cylindrical column of a viscous liquid of radius 5 cm and travels at a steady rate of 2.1 cm per sec. If the density of the liquid is 1.47 g per cc, find its viscosity. Assume $\mathrm{g}=980 \mathrm{~cm} / \mathrm{sec}^{2}$ and neglect the density of air.
(a) 2.65 poise
(b) 3.63 poise
(c) 1.52 poise
(d) 0.62 poise

Solution: Here due to force of buoyancy the bubble will move up and so viscous force which opposes the motion will act downward and as weight of bubble is zero, in dynamic equilibrium, upth. = F,
i.e., $\quad \frac{4}{3} \pi r^{3} \sigma g=6 \pi \eta r v_{T}$
or $\quad \eta=\frac{2}{9} \frac{\sigma r^{2} g}{v_{T}}=\frac{2}{9} \times \frac{1.47 \times(0.1)^{2} \times 980}{2.1}$
i.e., $\quad \eta=1.524$ poise

Answer (C)
Example- Two equal drops of water are falling through air with a steady velocity $v$. If the drops coalesced, what will be the new velocity?
(a) $2 \sqrt{2} v$
(b) $2 v$
(c) $2^{2 / 3} v$
(d) $2^{4 / 3} v$

Solutions: Let $r$ be the radius of each drop. The terminal velocity $v_{T}$ of a drop of radius $r$ is given by
$v_{T}=\frac{2}{9} \frac{r^{2}(\rho-\sigma) g}{\eta}$
Now when two drops each of radius r coalesce to form a new drop, the volume of coalesced drop will be given by
$\frac{4}{3} \pi R^{3}=\frac{4}{3} \pi r^{3}+\frac{4}{3} \pi r^{3}$
So the radius of the coalesced drop will be
$R=(2)^{1 / 3} r$
Hence, the new terminal velocity of the coalesced drop
$v_{T}^{\prime}=\frac{2}{9} \frac{\left[(2)^{1 / 3} r\right]^{2}(\rho-\sigma) g}{\eta}$
So dividing Eqn. (2) by (1)
$\frac{v_{T}^{\prime}}{v_{T}}=(2)^{2 / 3}$
Or $\quad v_{\mathrm{T}}=(2) 2 / 3 v \quad\left[\right.$ as $\left.v_{\mathrm{T}}=v\right]$
Answer (C)

