Chapter	Thermodynamics

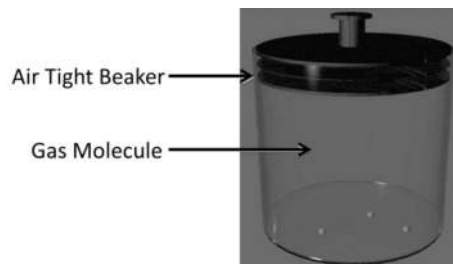**Day - 1**

THERMODYNAMICS SYSTEM

Any collection of large number of gas particle having certain values of P, V, T and U.
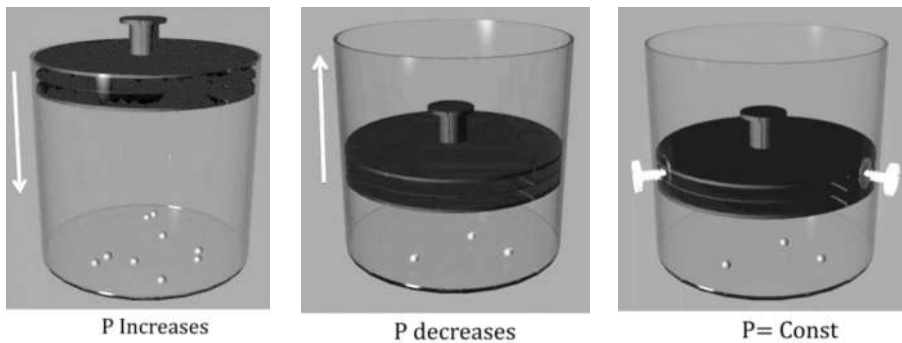

Air Tight Beaker

Gas Molecule

THERMODYNAMICS PROCESS

Any relation between thermodynamics variables

Eq. $PV^2 = T^3$, $U = PV$ etc.

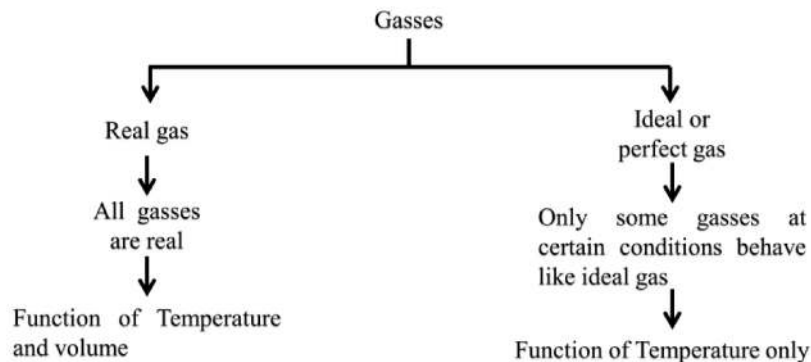Pressure $P = \dfrac{F}{A}$ Units, *Newton/met²* or Pascal

P Increases   P decreases   P= Const

VOLUME

Volume of container will be the volume of gas

5 mole

5 mole

INTERNAL ENERGY (U)

Gasses

Real gas → All gasses are real → Function of Temperature and volume

Ideal or perfect gas → Only some gasses at certain conditions behave like ideal gas → Function of Temperature only
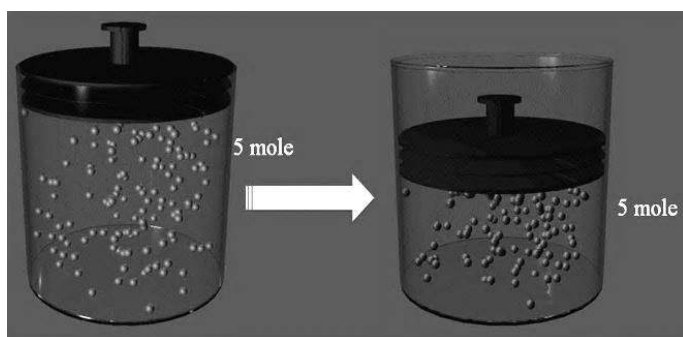
WORK DONE

W = F.ds

$\Rightarrow$ PAds

W = PdV

(i)   If P = constant

W = PdV

W = PΔV

W = P (V$_f$ -V$_i$)

(ii)   If P = variable

$$W = \int PdV$$

(iii) Graphically

Area under the P –V diagram

P = const.
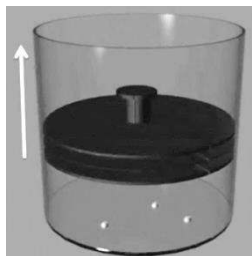W = PV

V = const.
W = 0
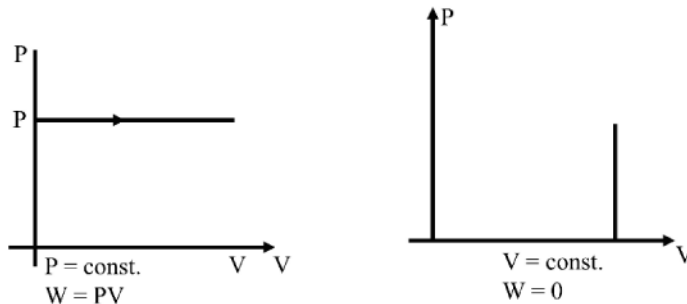
IF

V$_f$> V$_i$$\rightarrow$ expansion

W $\rightarrow$ +

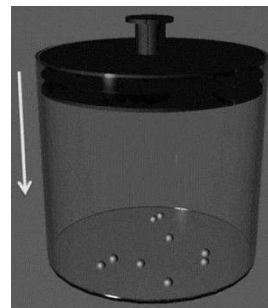We done by the gas
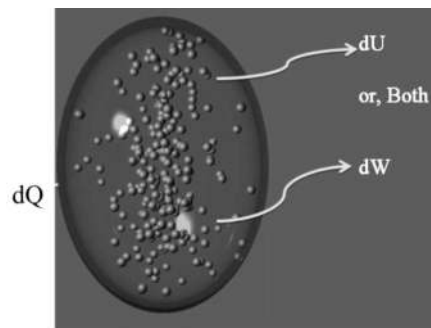
IF

$V_f < V_i \rightarrow$ Compression

$W \rightarrow -$

Work done on the gas

First law of Thermodynamics

dQ = dU + dW

Thermodynamics system

Sign convention

dQ = + if heat is given to system

     - if heat is taken from the system

dW = + if volume increases or expansion

     -if volume decrease or Compression

dU = + if Temperature increase

     - if Temperature decrease

Specific Heat

dQ = nCdT

$$C = \frac{dQ}{ndT}$$

Amount of heat required for 1 mole of gas to raise the temperature by 1^0C

Types of specific heat

(1) Specific heat at constant pressure (C_p)

(2) Specific heat at Constant Volume (C_v)

Mayer's formula $C_p - C_v = R$

Also $\gamma = \frac{C_p}{C_v}$

Here γ is constant depends on type of gas i.e. mono atomic Di atomic and Poly atomic
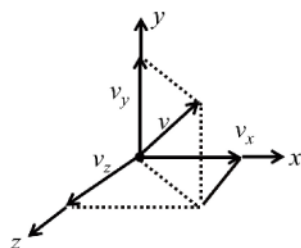
Now $C_p - C_v = R$

$$\gamma = \frac{C_p}{C_v}$$
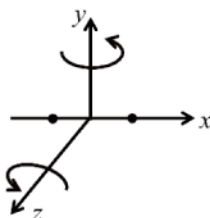
$\gamma C_v - C_v = R$

$$C_v = \frac{R}{\gamma - 1}$$
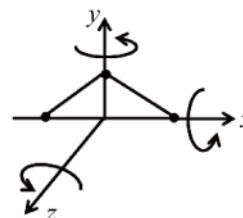
Also $C_p = \gamma\, C_v = \frac{\gamma R}{\gamma - 1}$

DEGREE OF FREEDOM

Mono atomic   Di atomic   Tri atomic

Degree of Freedom for different gases					
Atomicity of gas	Example	A	B	$f = (3A-B)$	Figure
Monoatomic	He, Ne, Ar	1	0	$f = 3$	•A
Diatomic	H_2, O_2, N_2, Cl_2 etc.	2	1	$f = 5$	A •······B······• B
Triatomic non linear	H_2O	3	3	$f = 6$	
Triatomic linear	$CO_2, BeCl_2$	3	2	$f = 7$	A•····B····•A····B····•A

Types of Thermodynamics Process
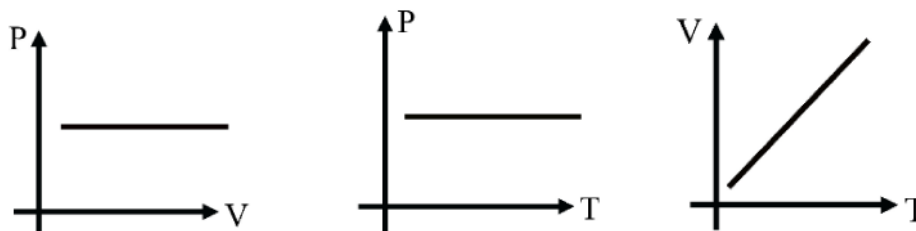
(1) Isobaric Process

P = Const. $\Delta P = 0$

By ideal gas equation

Pv = nRT

$V \propto T$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

GRAPHICALLY

Work done

dW = P dV

Ist Law of thermodynamics:

W = P ΔV

dQ = dU + dW {dU = nC$_V\Delta$T}

dQ = nC$_v\Delta$T + P ΔV {P ΔV = nRΔT}

$\Rightarrow$ nC$_v\Delta$T + nRΔT

$\Rightarrow$ n ΔT (C$_v$ + R)

dQ = nC$_p\Delta$T

(2) Isochoric Process

V = Const. ΔV = 0 $\Rightarrow$ W = 0

By ideal gas equation PV = nRT

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

P $\propto$ T

GRAPHICALLY

By 1st Law

dQ = dU + dW

V = Const. ΔV = 0,   dW = 0

dQ = dU {dU = nC$_V\Delta$T}

Isothermal Process

T = Const. ΔT = 0

U = Const. dU =0

PV =nRT

$\Rightarrow$ PV = Const. (K)

P$_1$V$_1$ =P$_2$V$_2$

Slow Process

Conducting Vessel

Ideal or Perfect Gas

dQ = dU + dW

dQ = dW

$$\int dW = \int P dV$$

$$\Rightarrow K \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\int dW = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

W = nRT$ln\frac{V_2}{V_1}$

W = nRT$ln\frac{P_1}{P_2}$

Also

W = 2.303 nRTlog$_{10}\frac{V_2}{V_1}$

W = 2.303 nRTlog$_{10}\frac{P_1}{P_2}$

GRAPHICALLY

(4) Adiabatic Process

dQ = 0, Q = Const.

PV$^\gamma$ = Const.

PV = nRT

$$P = \frac{nRT}{V}$$

$$\frac{nRT}{V} V^\gamma = Const.$$

TV$^{\gamma-1}$ = const.

Also

PV =nRT

$$V = \frac{nRT}{P}$$

$$P \left(\frac{nRT}{P}\right)^\gamma = Const.$$

P$^{1-\gamma}$T$^\gamma$ = Const.

Quick Process

Insulated Vessel

Ideal or perfect gas

1st Law

dQ = dU + dW

0 = dU + dW

dW = -dU

Now

$$dW = -nC_V \Delta T \left\{ C_V = \frac{R}{\gamma - 1} \right\}$$

$$\Rightarrow -\frac{nR}{\gamma - 1} \Delta T$$

$$\Rightarrow \frac{nR\Delta T}{1 - \gamma}$$

$$W = \frac{nR(T_f - T_i)}{1 - \gamma}$$

$$W = \frac{P_f V_f - P_i V_i}{1 - \gamma}$$

(5) Poly tropic Process

PV^x = Const. = K

$x \neq \gamma \quad x \neq 1$

$P_1 V_1^x = P_2 V_2^x = K$

$$W = \int P \, dV$$

$$\Rightarrow K \int_{V_1}^{V_2} V^{-x} \, dV$$

$$\Rightarrow \left[\frac{V^{-x+1}}{-x + 1} \right]_{V_1}^{V2}$$

$$\Rightarrow \left[\frac{P_2 V_2^x V_2^{-x+1} - P_1 V_1^x V_1^{-x+1}}{1 - x} \right]$$

$$W = \frac{P_2 V_2 - P_1 V_1}{1 - x}$$

$$W = \frac{nR\Delta T}{1 - x}$$

Specific Heat

dQ = nCΔT

$$C = \frac{dQ}{n\Delta T} \{ dQ = dU + dW \}$$

$$C = \frac{dU + dW}{n\Delta T}$$

$$C = \frac{dU}{n\Delta T} + \frac{dW}{n\Delta T}$$

$$C = \frac{nC_V \Delta T}{n\Delta T} + \frac{nR\Delta T}{n\Delta T(1-x)}$$

$$C = C_V + \frac{R}{1-x}$$

Difference in gram –molecular heats (C_p - C_v) of an ideal Gas:
Mayer's formula
Let T be the Kelvin temperature, P the pressure and V the volume of 1 mole of an ideal gas. Let this state of the gas be represented by a point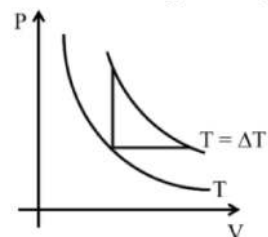 a on the pressure –volume curve at temperature T to T + ΔT, so that it reaches from state a to state c. from the first law of thermodynamics, the change in the internal energy of the gas in the process a → e is given by
$U_e - U_a = Q - W$

Where Q is the amount of heat taken by the gas and W is the work done by the gas since in this process the volume of the gas has remained constant (ΔV = 0), so W = P ΔV = O and Q = C, ΔT (by defined)
$$U_e - U_a = cv\Delta T$$
Now, suppose the gas is restored to its original state a, its temperature is again raised from T to T + ΔT but now at constant pressure, so that the gas reaches from state a to state b. The change in the internal energy of the gas in the internal energy of the gas in the pressure a →b is given by.
$$U_b - U_a = Q - W$$
But now Q = CpΔT and W = P ΔV (since the gas expanded from V to V+ ΔV at pressure)P.
$U_b - U_a = C_v\Delta T - P \Delta V$
In the initial state the volume of the gas V is and the temperature is T, while in the final state b the volume is (V + ΔV) and the temperature is (T + ΔT). Pressure P remains constant.
Hence for these state the gas equations are
PV = RT
P(V + ΔV) = RT(T + ΔT)
P ΔV = R ΔT
$U_b - U_a = C_p\Delta T - R\Delta T$
$U_e - U_a = U_b - U_a$
$C_v\Delta T = C_p\Delta T - R\Delta T$

$C_v = C_p - R$

$C_P - C_v = R$

Case-1 Conversion of graph

Pressure versus temperature (p.T) graph of n moles of an ideal gas. Plot the corresponding.

(a) Density verus volume (ρ-V) graph,

(b) Pressure verus volume (p-V) graph and

(c) Density verus pressure (ρ-p) graph.

Pressure A-B is an isothermal process

i.e.   T = constant

Hence, $p \propto \dfrac{1}{V}$

Or p-V graph will be a rectangle hyperbola with increasing p and decreasing V.

$p \propto \dfrac{1}{V}$. Hence, ρ-V graph is also a rectangular hyperbola with decreasing V and hence increasing ρ.

$p \propto p \qquad \left[\rho = \dfrac{pM}{RT}\right]$

Hence, ρ-p graph will be a straight line passing through origin, with increasing ρ and p.

Process B-C is an isochoric process , because p-T graph is a straight line passing through origin i.e. V= constant

Hence, p-V graph will be a straight line parallel to p-axis with increasing p.

Since, V = constant hence ρ will also be constant hence ρ-V graph will be a dot.

ρ-p graph will be a straight line parallel to p-axis with increasing p, because

ρ = constant

Process C-D is inverse of A-B and D-A is inverse of B-C.

Different values of p, V, T and ρ in tabular form are shown below.

	P	V	T	ρ
A	P_0	V_0	T_0	ρ_0
B	$2p_0$	$\dfrac{V_0}{2}$	T_0	$2\rho_0$
C	$4p_0$	$\dfrac{V_0}{2}$	$2T_0$	$2\rho_0$
D	$2p_0$	V_0	$2T_0$	ρ_0

$Here, V_0 = nR \left(\dfrac{T_0}{P_0}\right) and \rho_0 = \dfrac{P_0 M}{RT_0}$

The corresponding graphs are as follows

Case -2 finding efficiency by graph

Two moles of a monatomic ideal gas undergo a cyclic process ABCDA as shown in figure. BCD is a semicircle. Find the efficiency of the cycle.

Process AB is isochoric (V = constant).

Hence,

$\Delta W_{AB} = 0$

$\Delta W_{BCD} = p_0 V_0 + \frac{\pi}{2} (p_0) \left(\frac{V_0}{2} \right)$

$= \left(\frac{\pi}{4} + 1 \right) p_0 V_0$

$\Delta W_{DA} = -\frac{1}{2} \left(\frac{p_0}{2} + p_0 \right) (2V_0 - V_0)$

$= -\frac{3}{4} p_0 V$

$\Delta U_{AB} = nC_V \Delta T = (2) \left(\frac{3}{2} R \right) (T_B - T_A) \quad \left(n = 2, \ C_V = \frac{3}{2} R \right)$

$= 3R \left(\frac{p_0 V_0}{2R} - \frac{p_0 V_0}{nR} \right)$

$= \frac{3}{4} p_0 V_0 = \Delta Q_{AB} \quad \left(T = \frac{pV}{nR} \right)$

$\Delta U_{BCD} = nC_V \Delta T = (2) \left(\frac{3}{2} R \right) (T_D - T_B)$

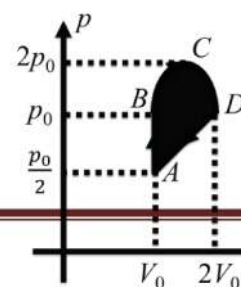$= (3R) \left(\frac{2p_0 V_0}{2R} - \frac{p_0 V_0}{2R} \right) \frac{3}{4} p_0 V_0$

Hence, $\Delta Q_{BCD} = \Delta U_{BCD} + \Delta W_{BCD}$

$= \left(\frac{\pi}{4} + \frac{5}{2} \right) p_0 V_0$

$\Delta U_{DA} = nC_V \Delta T$

$= (2) \left(\frac{3}{4} R \right) (T_A - T_D)$

$= (3R) \left(\frac{p_0 V_0}{4R} - \frac{2p_0 V_0}{2R} \right)$

$= -\frac{9}{4} p_0 V_0$

$\therefore \ \Delta Q_{DA} = \Delta U_{DA} + \Delta W_{DA}$

$= \frac{9}{4} p_0 V_0 - \frac{3}{4} p_0 V_0$

$= -3 p_0 V_0$

Net work done is,

$W_{net} = \left(\frac{\pi}{4} + 1 - \frac{3}{4} \right) p_0 V_0$

$= 1.04\ p_0 V_0$

and heat absorbed is

$Q_{ab} = \Delta Q_{+ve}$

Case -3 Example of first law

The density (ρ) verus pressure (p) graph of one mole of an ideal monoatomic gas undergoing a cyclic process is shown in figure. Molecular mass of gas is M.

(a) Find work done in each process.

(b) Find heat rejected by gas in one complete cycle.

(c) Find the efficiency of the cycle.

For Process 1-2 $\rho \propto$ p

$\therefore \qquad \dfrac{1}{V} \propto p$

$\therefore$ Process is isothermal.

$\Delta U_1 = 0$

$Q_1 = W_1 = nRT\ In\ \left(\dfrac{p_i}{p_f}\right)$

$= \dfrac{p_0 M}{\rho_0} In\ \left(\dfrac{1}{2}\right)\ \left(as\ n = 1\ and\ RT = \dfrac{PM}{\rho}\right)$

$= -\dfrac{p_0 M}{p_0} In\ (2)$

For process 2-3

$Q_2 = Q_p = n\ C_p \Delta T$

$= (1)\left(\dfrac{5}{2}R\right)(T_3 - T_2)$

$= \dfrac{5}{2}\left(\dfrac{p_3 M}{\rho_3} - \dfrac{p_2 M}{\rho_2}\right)$

$$= \frac{5}{2}\left(\frac{2p_0 M}{\rho_0} - \frac{2p_0 M}{2\rho_0}\right)$$

$$= 2.5\frac{p_0 M}{\rho_0}$$

$$\Delta U_2 = nC_V \Delta T$$

Substituting the values like above we get,

$$\Delta U_2 = \frac{1.5\rho_0 M}{\rho_0}$$

$$W_3 - Q_2 - \Delta U_2 = \frac{p_0 M}{\rho_0}$$

Substituting the values like above we get,

$$\Delta U_2 = \frac{1.5\rho_0 M}{\rho_0}$$

$$W_3 - Q_2 - \Delta U_2 = \frac{p_0 M}{\rho_0}$$

For Process 3-1

Density is constant. Hence,

Volume is constant.

$$\therefore \ W_3 = 0$$

$$\therefore \ Q_3 = \Delta U_2 = nC_V \Delta T$$

$$= (1)\left(\frac{3}{2}R\right)(T_1 - T_3)$$

$$= \frac{3}{2}\left(\frac{p_0 M}{\rho_0} - \frac{2p_0 M}{\rho_0}\right)$$
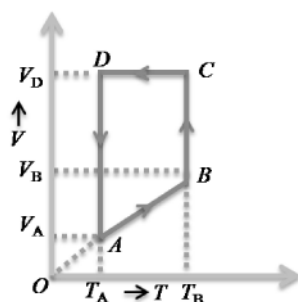
$$= \ -\frac{1.5 p_0 M}{\rho_0}$$

$$(b) \ \Sigma Q_{-ve} = |Q_1 + Q_3|$$

$$= \frac{p_0 M}{\rho_0}\left(\frac{3}{2} + In\ 2\right)$$

$$(c) \ \eta = \frac{W_{net}}{\Sigma Q_{+ve}} = \frac{(+p_0 M/\rho_0) + \left(\frac{-p_0 M}{\rho_0}In2\right)}{(2.5\ p_0 M/\rho_0)}$$

$$= \frac{2}{5}(1 - In2)$$

Example. A monatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratio are $\dfrac{V_B}{V_A} = 2$ and $\dfrac{V_D}{V_A} = 4$. If the temperature T_B at A is 27°C

Calculate

(a) the temperature of the gas at point B,

(b) heat absorbed or released by the gas in each process,

(c) the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R. **(Adv. 2001)**

Solution:- Given,

Number moles, n = 2

$$C_V = \frac{3}{2}R \text{ and } C_p = \frac{5}{2}R \qquad \text{(monoatomic)}$$

$$T_A = 27°C = 300 \text{ K}$$

Let $V_A = V_0$ then $V_B = 2V_0$

and $V_D = V_C = 4V_0$

(a) Process A→B

$$V \propto T$$

$$\Rightarrow \quad \frac{T_B}{T_A} = \frac{V_B}{V_A}$$

$$\therefore \quad T_B = T_A \left(\frac{V_B}{V_A}\right) = (300)(2) = 600 \text{ K}$$

(b) Process A→B

$$V \propto T$$
$$\Rightarrow \quad p = \text{constant}$$
$$\therefore \quad Q_{AB} = nC_p dT = nC_p(T_B - T_A)$$
$$= (2)\left(\frac{5}{2}R\right)(600 - 300)$$

$Q_{AB} = 1500 \text{ R}$ (absorbed)

Process B → C

$$T = \text{constant}$$
$$\therefore \quad dU = 0$$
$$\therefore \quad Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right)$$
$$= (2)(R)(600)\ln\left(\frac{4V_0}{2V_O}\right)$$
$$= (1200 \text{ R})\ln(2) = (1200 \text{ R})(0.693)$$

or $Q_{BC} = 831.6 \text{ R}$ (absorbed)

Process C→ D V = constant

$$\therefore \quad Q_{CD} = nC_V dT = nC_V(T_D - T_C)$$
$$= n\left(\frac{3}{2}R\right)(T_A - T_B)$$
$$(T_D = T_A \quad \text{and} \quad T_C = T_B)$$
$$= (2)\left(\frac{3}{2}R\right)(300 - 600)$$
$$Q_{CD} = -900R(\text{released})$$

Process D→ A T = constant

$$\Rightarrow \quad \Delta U = 0$$
$$\therefore \quad Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{V_A}{V_D}\right)$$
$$= (2)(R)(300)\ln\left(\frac{V_0}{4V_0}\right)$$
$$= 600 \text{ R} \ln\left(\frac{1}{4}\right)$$
$$Q_{DA} \approx -831.6 \text{ R (released)}$$

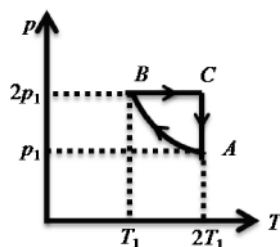**(c) In the complete cycle** $\Delta U = 0$

Therefore, from conservation of energy

$$W_{net} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$
$$W_{net} = 1500 \text{ R} + 831.6 \text{ R} - 900 \text{ R} - 831.6 \text{ R}$$

or $W_{net} = W_{total} = 600\,R$

Example. two moles of an ideal monatomic gas is taken through a cycle ABCA as shown in the p-T diagram. During the process AB, pressure and temperature of the gas such that pT = constant. If $T_1 = 300$ K, calculate (**Adv. 2000)**

(a) the work done on the gas in the process AB and
(b) the heat absorbed or released by the gas in each of the processes.
     Give answer in terms of the gas constant R.

Solution:- Number of moles, $n = 2, T_1 = 300$ K
During the process $A \rightarrow B$
pT = constant or p^2V = constant = K (say)

$$\therefore \quad p = \frac{\sqrt{K}}{\sqrt{V}}$$

$$\therefore \quad W_{A \rightarrow B} = \int_{V_A}^{V_B} p.\,dV = \int_{V_A}^{V_B} \frac{\sqrt{K}}{\sqrt{V}}\,dV$$

$$= 2\sqrt{K}[\sqrt{V_B} - \sqrt{V_A}]$$

$$2[\sqrt{KV_B} - \sqrt{KV_A}]$$

$$= 2[\sqrt{(p_B^2 V_B)V_B} - \sqrt{(p_A^2 V_A)V_A}]$$

$$= 2[p_B V_B - p_A V_A]$$

$$= [nRT_B - nRT_A] = 2\,nR[T_1 - 2T_1]$$

$$(2)(2)(R)[300 - 600] = -1200\,R.$$

$\therefore$ Work done on the gas in the process AB is 1200 R.
Alternation solution

$$pV = nRT$$
$$\therefore \quad pdV + Vdp = nRdT$$

or $\quad pdV + \left(\dfrac{nRT}{p}\right).dp = nRdT \qquad$ …(i)

From the given condition

$$pT = constant$$
$$pdT + Tdp = 0 \qquad \text{…(ii)}$$

From Eqs. (i) and (ii), we get

$$pdV = 2nRdT$$

$$\therefore W_{A \to B} = \int pdV = 2nR \int_{T_A}^{T_B} dT = 2nR(T_B - T_A)$$

$$= 2nR(T_1 - 2T_1) = (2)(2)(300 - 600)$$

or $W_{A \to B} = -1200\ R$

(b) **Heat absorbed/released in different processes.**

Since, the gas is monatomic.

Therefore, $C_V = \dfrac{3}{2}R$ and $C_p = \dfrac{5}{2}$ and $\gamma = \dfrac{5}{3}$

Process A→B $\Delta U = nC_V \Delta T$

$$= (2)\left(\dfrac{3}{2}R\right)(T_B - T_A)$$

$$= (2)\left(\dfrac{3}{2}R\right)(300 - 600) = -900\ R$$

$$\therefore \quad Q_{A \to B} = W_{A \to B} + \Delta U$$
$$= (-1200\ R) - (900\ R)$$

$Q_{A \to B} = -2100\ R \qquad$ (released)

Alternation solution

In the process $pV^x = constant$

Molar heat capacity, $C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1 - x}$

Here the process is $p^2V = constant$

or $\quad pV^{1/2} = constant$

i.e. $x = \frac{1}{2}$

$$\therefore \quad C = \frac{R}{\frac{5}{3} - 1} + \frac{R}{1 - \frac{1}{2}}$$

$$\therefore \quad C = 3.5\,R$$

$$\therefore \quad Q_{A \to B} = nC\Delta T$$

$$= (2)(3.5\,R)(300 - 600)$$

or $Q_{A \to B} = -2100\,R$

Process B $\to$ C Process is isobaric

$$Q_{B \to C} = nC_p \Delta T = (2)\left(\frac{5}{2}R\right)(T_C - T_B)$$

$$= 2\left(\frac{5}{2}R\right)(2T_1 - T_1)$$

$$= (5R)(600 - 300)$$

$$Q_{B \to C} = 1500\,R\,(absorbed)$$

Process C $\to$ A Process is isothermal.

$$\therefore \quad \Delta U = 0$$

and $Q_{C \to A} = W_{C \to A} = nRT_C \ln\left(\frac{p_C}{p_A}\right)$

$$= nR(2T_1)\ln\left(\frac{2p_1}{p_1}\right) = (2)(R)(600)\ln(2)$$

$$Q_{C \to A} = 831.6\,R\,(absorbed)$$

NOTE

In first law of thermodynamics, $(dQ = dU + dW)$ we come across three terms dQ, dU and dW.

$dU = nC_V dT$ for all the processes whether it is isobaric isochoric or else and $dQ = nCdT$ where

$$C = \frac{R}{r - 1} + \frac{R}{1 - X}$$

In the process pV^r= constant.

In both terms we require $dT(= T_f - t_i)$ only. The third term dW is obviously $dQ - dU$. Therefore if any process change in temperature (dT) and p-V relation is known, then the above method is the simplest one. Note that even if we have V-T or T-p relation, it can be converted into p-V relation by the equation $pV = nRT$

Example. One mole of an ideal monatomic gas is taken round the cyclic process ABCA as shown in figure. Calculate **(Adv. 1998)**

(a) the work done by the gas.
(b) The heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB.
(c) the net heat absorbed by the gas in the path BC.
(d) the maximum temperature attained by the gas during the cycle.

*solution:-*ABCA is a clockwise cyclic process.

∴ Work done by the gas

$$W = +\text{Area of triangle ABC}$$

$$= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2V_0 - V_0)(3p_0 - p_0)$$

$$W = p_0V_0$$

(b) Number of moles n = 1 and gas is monatomic, therefore

$$C_V = \frac{3}{2}R \quad \text{and} \quad C_p = \frac{5}{2}R$$

$$\Rightarrow \frac{C_V}{R} = \frac{3}{2} \quad \text{and} \quad \frac{C_p}{R} = \frac{5}{2}$$

(i)**Heat rejected in path CA** (process is isobaric)

$$\therefore \quad dQ_{CA} = C_p dT = C_p(T_f - T_i)$$

$$= C_p\left(\frac{p_f V_f}{R} - \frac{p_i V_i}{R}\right) = \frac{C_p}{R}(p_f - p_i V_i)$$

Substituting the values

$$dQ_{CA} = \frac{5}{2}(p_0V_0 - 2p_0V_0) = -\frac{5}{2}p_0V_0$$

Therefore, heat rejected in the process CA is $\frac{5}{2}p_0V_0$

(ii) **Heat absorbed in path AB** (process is isochoric)

$$\therefore \quad dQ_{AB} = C_V dT = C_V(T_f - T_i)$$

$$= C_V\left(\frac{p_f V_f}{R} - \frac{p_i V_i}{R}\right)$$

$$\frac{C_V}{R}(p_f V_f - p_i V_i)$$

$$\frac{3}{2}(p_f V_f - p_i V_i)$$

$$= \frac{3}{2}(3p_0 V_0 - p_0 V_0)$$

$$dQ_{AB} = 3p_0 V_0$$

$\therefore$ Heat absorbed in the process AB is $3\,p_0 V_0$

(c) Let dQ_{BC} be the heat absorbed in the process BC

Total heat absorbed,

$$dQ = dQ_{CA} + dQ_{AB} + dQ_{BC}$$

$$dQ = \left(-\frac{5}{2}P_0 V_0\right) + (3p_0 V_0) + dQ_{BC}$$

$$dQ = dQ_{BC} + \frac{p_0 V_0}{2}$$

Change in internal energy, $dU = 0$

$$\therefore dQ = dW$$

$$\therefore \quad dQ_{BC} + \frac{p_0 V_0}{2} = p_0 V_0 \quad \therefore dQ_{BC} = \frac{p_0 V_0}{2}$$

$\therefore$ Heat absorbed in the process BC is $\frac{p_0 V_0}{2}$

(c) Maximum temperature of the gas will be somewhere between B and C. Line BC is a straight line. Therefore p-V equation for the process BC can be written as

$$p = -mV + c(y = mx + c)$$

Here, $m = \frac{2p_0}{V_0}$ and $c = 5p_0$

$$\therefore \quad p = -\left(\frac{2p_0}{V_0}\right)V + 5\,p_0$$

Multiplying the equation by V,

$$pV = -\left(\frac{2p_0}{V_0}\right)V^2 + 5\,p_0 V \qquad\qquad (pV = RT \text{ for } n = 1)$$

$$RT = -\left(\frac{2p_0}{V_0}\right)V^2 + 5\,p_0 V$$

or $T = \frac{1}{R}\left[5p_0 V - \frac{2p_0}{V_0}V^2\right]$
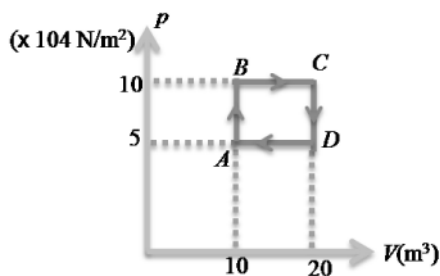
For T to be maximum, $\frac{dT}{dV} = 0$

$$\Rightarrow \quad 5p_0 - \frac{4p_0}{V_0}V = 0 \Rightarrow V = \frac{5V_0}{4}$$

i.e. at $V = = \frac{5V_0}{4}$, (on line BC), temperature of the gas is maximum. From Eq. (i) this maximum temperature will be

$$T_{max} = \frac{1}{R}\left[5p_0\left(\frac{5V_0}{4}\right) - \frac{2p_0}{V_0}\left(\frac{5V_0}{4}\right)^2\right]$$

$$T_{max} = \frac{25}{8}\frac{p_0V_0}{R}$$

Example. A sample of 2 kg monoatomic helium (assumed ideal) is taken through the process ABC and another of 2 kg of the same gas is taken through the process ADC (see fig). (**Adv. 1997**)

(a) What is the temperature of helium in each of the states A, B, C and D?
(b) Is there any way of telling afterwards which sample of helium went through the process ABC and which went through the process ADC? Write Yes or NO.
(c) How much is the heat involved in the process ABC and ADC?

*Solution:-*number of gram moles of He,

$$n = \frac{m}{M} = \frac{2 \times 10^3}{4} = 500$$

(a)$V_A = 10 \text{ m}^3, P_A = 5 \times 10^4 \text{N/m}^2$

$$\therefore \quad T_A = \frac{p_A V_A}{nR} = \frac{(10)(5 \times 10^4)}{(500)(8.31)}K$$

or $T_A = 120.34 \text{ K}$
Similarly, $V_B = 10 \text{ m}^3, P_B = 10 \times 10^4 \text{N/m}^2$

$$\therefore \quad T_B = \frac{(10)(10 \times 10^4)}{(500)(8.31)}K$$

$$\therefore T_B = 240.68 \text{ K}$$

$V_C = 20 \text{ m}^3, p_C = 10 \times 10^4 \text{N/m}^2$

$$T_C = \frac{(20)(10 \times 10^4)}{(500)(8.31)}k$$

$$T_C = 481.36 \text{ K}$$

and $V_D = 20\text{m}^3$, $P_D = 5 \times 10^4 \text{N/m}^2$

$$\therefore \quad V_D = \frac{(20)(5 \times 10^4)}{(500)(8.31)} \text{K}$$

$$T_D = 240.68 \text{ K}$$

(b) No, it is not possible to tell afterwards which sample went through the process ABC or ADC . but during the process if we note down the work done in both the processes, then the process which require more work goes through process ABC.

(c) In the process ABC

$$\Delta U = nC_V \, \Delta T = n\left(\frac{3}{2}R\right)(T_C - T_A)$$

$$= (500)\left(\frac{3}{2}\right)(8.31)(481.36 - 120.34)\text{J}$$

$$\Delta U = 2.25 \times 10^6 \text{ J}$$

$$\text{and} \Delta W = \text{Area under BC}$$

$$= (20 - 10)(10) \times 10^4 \text{J} = 10^6 \text{ J}$$

$$\therefore \quad \Delta Q_{ABC} = \Delta U + \Delta W = (2.25 \times 10^6 + 10^6)\text{J}$$

$$\Delta Q_{ABC} = 3.25 \times 10^6 \text{ J}$$

In the process ADC ΔU will be same (because it depends on initial and final temperature only)

$$\Delta W = \text{Area under AD}$$

$$= (20 - 10)(5 \times 10^4)\text{j} = 0.5 \times 10^6 \text{ J}$$

$$\Delta Q_{ADC} = \Delta U + \Delta W = (2.25 \times 10^6 + 0.5 \times 10^6)\text{J N}$$

$$\Delta Q_{ADC} = 2.75 \times 10^6 \text{J}$$