

Chapter

# **Heat Transfer**

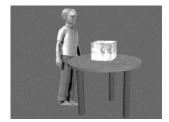
#### **Day - 1**

Transfer of energy from one place to other place

We know for heat or energy transfer a important factor is change in temperature ( $\Delta T$ ) In this case block appears to be more hot as compare to wood block If we feels

Cold  $\rightarrow$  we reject heat

Hot  $\rightarrow$  we receive heat



In this case Iron appears to be more hot as compare to wood block If we feels

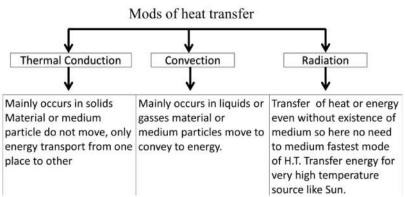
Hot  $\rightarrow$  we receive heat

Cold  $\rightarrow$  we reject heat



#### In winters

In cold countries glass window made of double or triple layers Also Eskimos made their huts by double or triple layers of ice Except  $\Delta T$  there is one more point called conductivity of heat





# **Thermal conduction**

Slowest mode of heat transfer.



# **CONVECTION**

In liquids



In gasses



## Radiation

Transfer of heat or energy even without existence of medium. So here no need of medium



## Thermal conduction

In steady state

$$\frac{dQ}{dt} = H$$

Rate of transfer of heat or heat current





$$\frac{\frac{dQ}{dt} \alpha A}{\frac{-dT}{dx}}$$

$$\frac{dQ}{dt} = -K. A \frac{dT}{dx}$$

Here  $\frac{dT}{dx}$  Temperature gradient negative always because

$$dT \uparrow$$
,  $dx \downarrow$   
 $dT \downarrow$ ,  $dx \uparrow$ 

and K is coefficient of thermal conductivity

$$K_{cu} \rightarrow 290 \text{ to } 320$$

$$K_{Fe} \rightarrow 80 \text{ to } 150$$

$$K_{wood} \rightarrow 0.7$$

$$K_{air} \rightarrow 0.005$$

Here air is very bad conductor of heat

# Thermal resistance

$$R = \frac{l}{KA}$$
If H =  $\frac{dQ}{dt} = KA\frac{\Delta T}{l}$ 

## **Combinations of slabs**

Series Combinations

Heat current through slab l

$$\frac{dQ}{dt} = H_1 = \frac{K_1 A (T_1 - T_C)}{d}$$

Heat T<sub>c</sub> is junction temperature

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \mathrm{H}_2 = \frac{\mathrm{K}_2 \mathrm{A} (\mathrm{T}_\mathrm{c} - \mathrm{T}_2)}{\mathrm{d}}$$

In series Heat current must be same

$$H_1 = H_2 \Rightarrow \frac{K_1 A (T_1 - T_C)}{d} = \frac{K_2 A (T_c - T_2)}{d}$$

$$K_1 \ T_1 - K_1 T_c = K_2 T_c - K_2 T_2$$

$$\label{eq:Junction Temperature Tc} \text{Junction Temperature T}_c = \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}$$

$$\mathbf{H}_{1} = \frac{\mathbf{K}_{1}\mathbf{A}}{\mathbf{d}} \Big( \mathbf{T}_{1} - \frac{\mathbf{K}_{1}\mathbf{T}_{1} + \mathbf{K}_{2}\mathbf{T}_{2}}{\mathbf{K}_{1} + \mathbf{K}_{2}} \Big)$$





$$\begin{split} H_1 &= \frac{K_1 A}{d} \bigg( \frac{K_1 T_1 + K_2 T_1 - K_1 T_1 - K_2 T_2}{K_1 + K_2} \bigg) \\ H_1 &= \frac{K_1 A}{d} \bigg( \frac{K_2 T_1 - K_2 T_2}{K_1 + K_2} \bigg) \\ &\Rightarrow \frac{K_1 K_2 A}{d} \bigg( \frac{T_1 - T_2}{K_1 + K_2} \bigg) \end{split}$$

Equivalent Conductivity of heat

$$H = \frac{K_{eq}A(T_1 - T_2)}{2d}$$

$$H = \frac{K_1K_2}{(K_1 + K_2)d}A(T_1 - T_2)$$

 $H = H_1$ 

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

#### (II) Parallel combination

Heat current through slab 1

$$H_{1} = \frac{dQ_{1}}{dt} = \frac{K_{1}A(T_{1} - T_{2})}{d}$$

$$H_{2} = \frac{dQ_{2}}{dt} = \frac{K_{2}A(T_{1} - T_{2})}{d}$$

$$H_{eq} = H_1 + H_2$$

$$\begin{split} \frac{K_{\rm eq} 2A(T_1 - T_2)}{d} &= \frac{K_1 A(T_1 - T_2)}{d} + \frac{K_2 A(T_1 - T_2)}{d} \\ & \left(K_{\rm eq} = \frac{K_1 + K_2}{2}\right) \end{split}$$



# Interaction of radiation with matter

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

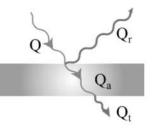
$$(1) Q = Q_a + Q_r + Q_t$$

$$(2) \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t = 1$$

$$(3) a = \frac{Q_a}{Q} = \text{Absorptance or absorbing power}$$

$$r = \frac{Q_r}{Q} = \text{Reflectance or reflecting power}$$

$$t = \frac{Q_t}{Q} = \text{Transmittance or transmitting power}$$



- (4) r, a and t all are the pure ratios so they have no unit and dimension.
- (5) Different bodies



- (i) If a = t = 0 and  $r = 1 \rightarrow body$  is perfect reflector
- (ii) If r = t = 0 and  $a = 1 \rightarrow$  body is perfectly black body
- (iii) If a = r = 0 and  $t = 1 \rightarrow body$  is perfect transmitter
- (iv) If  $t = 0 \Rightarrow r + a = 1$  or a = 1- r, i.e, good reflectors are bad absorbers.

# Emissive power, absorptive power and emissivity

If temperature of a body is more than it's surrounding then body emits thermal radiation.

(1) Monochromatic Emittance or Spectral emissive power ( $e_{\lambda}$ ): For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface with in a unit wavelength around  $\lambda$ , i.e., lying between  $\left(\lambda - \frac{1}{2}\right)$  to  $\left(\lambda + \frac{1}{2}\right)$ .

Spectral emissive power 
$$(e_{\lambda}) = \frac{Energy}{Area \times time \times wavelength}$$

Units: 
$$\frac{\text{Joule}}{\text{m}^2 \times \text{s} \times \text{Å}}$$
 and Dimension:  $[ML^{-1}T^{-3}]$ 

(2) **Total emittance or total emissive power (e)**: It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.  $e = \int_0^\infty e_{\lambda} d\lambda$ 

Unit: 
$$\frac{Joule}{m^2 \times s}$$
 or  $\frac{Watt}{m^2}$  and Dimension: [MT<sup>-3</sup>]

- (3) Monochromatic absorptance or spectral absorptive power  $(a_{\lambda})$ : It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unit less quantity. It is represented by  $a_{\lambda}$ .
- (4) **Total absorptance or total absorpting power (a):** It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

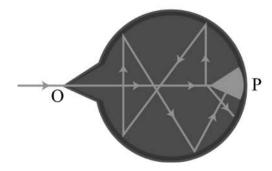
- (5) **Emissivity** ( $\varepsilon$ ): Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at the temperature, i.e.,  $\varepsilon = \frac{e}{E}$  ( $\varepsilon \rightarrow read\ as\ epsilon$ )
- (i) For perfectly black body  $\varepsilon = 1$
- (ii) For highly polished body  $\epsilon = 0$
- (iii) But for practical bodies emissivity ( $\epsilon$ ) lies between zero and one (0< $\epsilon$ < 1).

# Perfectly Black Body

(1) A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it .



- (2) As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity, i.e., t = 0 and  $r = 0 \Rightarrow a = 1$ .
- (3) We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.
- (4) When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.) it emits all possible radiation so it is an example of black body.



### Kirchhoff's law

According to this law the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that

temperature. Hence 
$$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \cdots = \left(\frac{E}{A}\right)_{\text{Perfectly black body}}$$

But for perfectly black body A = 1, i.e.,  $\frac{e}{a} = E$ 

If emissive and absorptive powers are considered for a particular wavelength  $\lambda$ ,  $\left(\frac{e_{\lambda}}{a_{\lambda}}\right) = (E_{\lambda})_{black}$ 

Now since  $(E_{\lambda})_{black}$  is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

### Stefan's law

According to it radiant energy emitted by a perfectly black body per unit area per sec (i.e., emissive power of black body) is directly proportional to the fourth power of its absolute temperature , i.e.,  $E \propto T^4 \Rightarrow E = \sigma T^4$ 

Whereois a constant called Stefan's constant having dimension [MT<sup>-3</sup> $\theta^{-4}$ ] and value 5.67 x  $10^{-8}$  W/ m<sup>2</sup>K<sup>4</sup>.

- (i) For ordinary body:  $e = \varepsilon E = \varepsilon \sigma T^4$
- (ii) Radiant energy: If Q is the total energy radiated by the ordinary body than

$$e = \frac{Q}{A \times t} = \varepsilon \sigma T^4 \Rightarrow Q = A \varepsilon \sigma T^4 t$$



(iii) Radiant power (P): It is defined as energy radiated per unit area, i.e.,

$$P = \frac{Q}{t} = A\varepsilon\sigma T^4.$$

(iv) If an ordinary body at temperature T is surrounded by a body at temperature  $T_0$ , then Stefan's law may be put as  $e = \varepsilon \sigma (T^4 - T_0^4)$ 

# Rate of loss of heat (r<sub>h</sub>) and rate of cooling (r<sub>c</sub>)

(1) Rate of loss of heat (or initial rate of loss of heat): In an ordinary body at temperature T is placed in an environment of temperature  $T_0$  ( $T_0 < T$ ) then heat loss by radiation is given by  $\Delta Q = Q_{emission} - Q_{absorption} = A\varepsilon\sigma(T^4 - T_0^4)t$ 

(2) Rate of loss of heat 
$$(R_H) = \frac{dQ}{dt} = A\epsilon\sigma(T^4 - T_0^4)$$

(i) If two bodies are made of same material, have same surface finish and are at the

same initial temperature then 
$$\frac{dQ}{dt} \propto A \Rightarrow \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{A_1}{A_2}$$

(3) **Initial rate of fall in temperature (Rate of cooling):** If m is the body and c is the specific heat then

$$\frac{dQ}{dt} = mc. \frac{dT}{dt} = mc\frac{d\theta}{dt} \quad (\because Q = mc\Delta T \text{ and } dT = d\theta)$$
(i) Rate of cooling  $(R_C) = \frac{d\theta}{dt} = \frac{(dQ/dt)}{mc} = \frac{A\epsilon\sigma}{mc} (T^4 - T_0^4)$ 

$$= \frac{A\epsilon\sigma}{Voc} (T^4 - T_0^4); \text{ where } m = \text{density } (\rho) \times \text{volume}(V)$$

(ii) for two bodies of the same material under identical environments , the ratio of their rate of cooling is  $\frac{(R_c)_1}{(R_c)_2} = \frac{A_1}{A_2} \cdot \frac{V_2}{V_1}$ 

# Newton 's law cooling

When the temperature difference between the body and its surrounding is not very large i.e.T  $-T_0 = \Delta T$  then  $T^4 - T_0^4$  may be approximated as  $4T_0^3 \Delta T$ 

By Stefan'slaw, 
$$\frac{dT}{dt} = \frac{A\epsilon\sigma}{mc} [T^4 - T_0^4]$$
  
Hence  $\frac{dT}{dt} = \frac{A\epsilon\sigma}{mc} 4T_0^3 \Delta T \Rightarrow \frac{dT}{dt} \propto \Delta T \text{ or } \frac{d\theta}{dt} \propto \theta - \theta_0$ 

i.e., if the temperature of body is not very different from surrounding, **rate of cooling is proportional to temperature difference** between the body and its surrounding. This law is called Newton's law of cooling.

(1) Greater the temperature difference between body and its surrounding greater will be the rate of cooling.



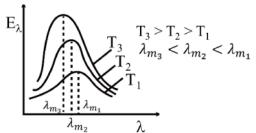
(2) If  $\theta = \theta_0$ ,  $\frac{d\theta}{dt} = 0$  i.e. a body can never be cooled to a temperature lesser than its surrounding by radiation.

(3) If a body cools by radiation from  $\theta_1^{\circ}C$  to  $\theta_2^{\circ}C$  in time t, then  $\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$  and  $\theta_{av} = \frac{\theta_1 + \theta_2}{2}$ . The Newton's law of cooling becomes  $\left[\frac{\theta_1 - \theta_2}{t}\right] = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$ .

This form of law helps in solving numericals.

# Wien's displacement law

According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, i.e.  $\lambda_m T = b = constant$  where b is Wien's constant and has value 2.89 x 10<sup>-3</sup> m-K. As the temperature of the body increases, the wavelength at which the spectral intensity ( $E_{\lambda}$ ) is maximum shifts towards left. Therefore it is also called Wien's displacement law.



This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding  $\lambda_m$  the temperature of the star T ( = b /  $\lambda_m$ ) is determined.

#### Temperature of the Sun and Solar Constant

If R is the radius of the sun and T its temperature, then the energy emitted by the sun per sec through radiation in accordance with Stefan's law will be given by

$$P = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

In reaching earth this energy will spread over a sphere of radius r ( = average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

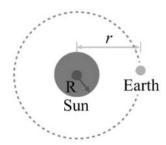
$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$i.e. \ T = \left[ \left( \frac{r}{R} \right)^2 \frac{S}{\sigma} \right]^{1/4}$$

$$= \left[ \left( \frac{1.5 \times 10^8}{7 \times 10^5} \right)^2 \times \frac{1.4 \times 10^3}{5.67 \times 10^{-8}} \right]^{1/4} \simeq 5800 \text{ K}$$

As  $r = 1.5 \times 10^8 \text{ km}$ ,  $R = 7 \times 10^5 \text{ km}$ ,





$$S=2~\frac{cal}{cm^2min}=1.4\frac{kW}{m^2}$$
 and  $\sigma=5.67\times 10^{-8}~\frac{W}{m^2K^4}$ 

This result is in good agreement with the experimental value of temperature of sun, i.e., 6000 K.

## Case 1 Growth of ice on ponds

$$\frac{dQ}{dt} = L_f \frac{dm}{dt}$$

$$\frac{\Delta T}{R} = Lf \frac{d}{dt} (A\rho y)$$

$$\frac{o - (-T)}{Y/KA} = Lf A \rho \frac{dy}{dt}$$

$$\int_0^t dt = \frac{\rho L f}{\kappa T} \int_0^y y \, dy$$

$$t = \frac{\rho L f}{2KT} y^2$$
 Indep. of area of pond

If thickness is increased from  $Y_1$  to  $Y_2$  then

$$t = \frac{PL}{KT} \int_{y_1}^{y_2} y \, dy \, p \frac{\rho L}{2KT} (y_2^2 - y_1^2)$$

$$t_1; t_2; t_3... = 1^2; 2^2; 3^2$$

$$\Delta t_1$$
;  $\Delta t_2$ ;  $\Delta t_3$  ... = 1; 3; 5

### Case 2 Radial heat flow

Therml resistance  $R = \frac{l}{KA}$ 

$$dR = \frac{dr}{K4\pi r^2}$$

$$\int_0^R dR = \frac{1}{4\pi K} \int_a^b r^{-2} dr$$

$$R = \frac{1}{4\pi k} \left( \frac{1}{a} - \frac{1}{b} \right)$$



$$\frac{dR}{dt} = H = \frac{\Delta T}{R}$$

$$= \frac{(T_1 - T_2)(4\pi k)}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$H = \frac{(T_1 - T_2)(ab)(4\pi k)}{b - a}$$

Case-3 Steam passing cylindrical pipe 
$$R = \frac{l}{KA} \implies \int dR = \int \frac{dr}{k2\pi r}$$

$$R = \frac{1}{2\pi K} \int_{a}^{b} \frac{dr}{r}$$

$$R = \frac{1}{2\pi k} \ln \frac{b}{a}$$

$$\frac{dQ}{dt} = H = \frac{\Delta T}{R} = \frac{(T_1 - T_2)2\pi k}{\ln\left(\frac{b}{a}\right)}$$

#### Case: 4 and Case 5

- 1. Cooling by conduction or radiation
- (i) By conduction A body P of mass m and specific heat C is connected to a large body Q (of specific heat infinite) through a rod of length l, thermal conductivity K and area of crosssection A. Temperature of Q is  $\theta_0$  ( $< \theta_i$ ). This temperature will remain constant as its specific heat is very high. Heat will flow P to Q through the rod. If we neglect the loss of heat due to radiation then due to this heat transfer, temperature of P will decrease but temperature of Q will remain almost constant. At time t, suppose temperature of P becomes  $\theta$  then due to temperature difference heat transfer through the rod.

$$\frac{dQ}{dt} = H = \frac{TD}{R} = \frac{\theta - \theta_0}{R} \qquad \dots (i)$$

Here, 
$$R = \frac{I}{KA}$$

Now, if we apply equation of calorimetry in P, then

$$Q = mc(-\Delta\theta)or \frac{dQ}{dt} = mc\left(-\frac{d\theta}{dt}\right) \qquad ...(ii)$$

Equating Eqs. (i) and (ii), we have

$$-\frac{d\theta}{dt} = \frac{TD}{mcR} = \frac{\theta - \theta_0}{mcR} = Rate \ of \ cooling \qquad ... (iii)$$

So, this is the rate of cooling by conduction.



or 
$$-\frac{d\theta}{dt} \propto TD$$
 ...  $(iv)$ 

### Cooling by Radiation

Consider a hot body at temperature T placed in an environment at a lower temperature  $T_0$ . The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations at a rate

$$P_1 = eA\sigma T^4$$

and is receiving energy by absorbing radiations at a rate

$$P_2 = aA\sigma T_0^4$$

Here, 'a' is a pure number between 0 and 1 indicating the relative ability of the surface to absorb radiation from its surroundings. Note that this 'a' is different from the absorptive power 'a'. In thermal equilibrium, both the body and the surroundings have the same temperature (say  $T_c$ ) and,

$$P_1 = P_2$$
 or  $eA\sigma T_c^4 = aA\sigma T_c^4$  or  $e = a$ 

Thus, when  $T > T_0$ , the net rate of heat transfer from the body to the surroundings is

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_0^4) \text{ or } mc\left(-\frac{dT}{dt}\right) = eA\sigma(T^4 - T_c^4)$$

Rate of cooling

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc} \left(T^4 - T_0^4\right) \quad or -\frac{dT}{dt} \propto \left(T^4 - T_0^4\right)$$

#### **Newton's Law of Cooling**

According to this law, if the temperature T of the body is not very different from that of the surrounding  $T_0$ , then rate of cooling  $-\frac{dT}{dt}$  is proportional to the temperature difference between them. To prove it, let us assume that

$$T = T_0 + \Delta T$$

So that 
$$T^4 = (T_0 + \Delta T)^4 = T_0^4 \left(1 + \frac{\Delta T}{T_0}\right)^4$$

$$\approx T_0^4 \left(1 + \frac{4\Delta T}{T_0}\right)$$
 (from binomial expansion)

$$Triangleright : (T^4 - T_0^4) = 4T_0^3 (\Delta T)$$

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or 
$$(T^4 - T_0^4) \propto \Delta T$$
 (as  $T_0 = constant$ )

Now, we have already shown that rate of cooling

$$\left(-\frac{dT}{dt}\right) \propto (T^4 - T_0^4)$$

and here we have shown that

$$(T^4 - T_0^4) \propto \Delta T$$

If the temperature difference is small.

Thus, rate of cooling

$$-\frac{dT}{dt} \propto \Delta T$$
 or  $-\frac{d\theta}{dt} \propto \Delta \theta$ 

as 
$$dT = d\theta$$
 or  $\Delta T = \Delta \theta$ 

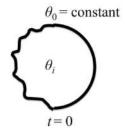
## Variation of Temperature of a Body According to Newton's Law

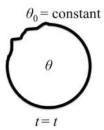
suppose a body has a temperature  $\theta_i$  at time t=0. It is placed in an atmosphere whose temperature is  $\theta_0$ . We are interested in finding the temperature of the body at time t. Assuming Newton's law of cooling to hold good or by assuming that the temperature difference is small. As per this law,

Rate of cooling ∝ temperature difference

or 
$$\left(-\frac{d\theta}{dt}\right) = \left(\frac{eA\sigma}{mc}\right) (4\theta_0^3) (\theta - \theta_0)$$

or 
$$\left(-\frac{d\theta}{dt}\right) = \alpha \left(\theta - \theta_0\right)$$





Here, 
$$\theta = \left(\frac{4eA\sigma\theta_0^3}{mc}\right)$$
 (is a constant)

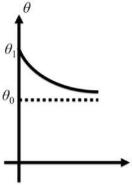
$$\therefore \qquad \qquad \int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -\alpha \int_0^t dt$$

$$\therefore \quad \theta = \theta_0 + (\theta_i - \theta_0)e^{-\alpha t}$$



From this expression we see that  $\theta = \theta_i$  at t = 0 and  $\theta = \theta_0$  at  $t = \infty$ , i.e. temperature of the body varies exponentially with time from  $\theta_i$  to  $\theta_0$  ( $< \theta_i$ ).

The temperature versus time graph



**Example:-**. A cylindrical block of length 0.4 m and area of cross –section 0.04 m<sup>2</sup> is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross –section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. if the thermal conductivity of the material of the cylinder is 10 W/mK and the specific heat capacity of the material of the disc is 600 J/kg –K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder. (Adv. 1992)

**Solution:** Let at any time temperature of the disc be  $\theta$ .

At this moment rate of heat flow.

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{1} = \frac{KA}{1}(\theta_0 - \theta) \qquad ....(i)$$

This heat is utilized in increasing the temperature of the disc.

Hence,

$$\frac{dQ}{dt} = ms \frac{d\theta}{dt} \qquad \qquad .....(ii)$$

Equating Eqs. (i) and (ii), we have

$$ms\frac{d\theta}{dt} = \frac{KA}{l}(\theta_0 - \theta)$$

Therefore,  $\frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl} dt$ 

or 
$$\int_{300 \text{ K}}^{350 \text{ K}} \frac{d\theta}{\theta_0 - \theta} = \frac{KA}{msl} \int_0^t dt$$
or 
$$[-\ln(\theta_0 - \theta)]_{300K}^{350K} = \frac{KA}{msl} t$$

$$t = \frac{msl}{KA} \ln \left( \frac{\theta_0 - 300}{\theta_0 - 350} \right)$$

I = 0.4 m  $\frac{dQ}{dt}$ 

Substituting the values, we have



$$t = \frac{(0.4)(600)(0.4)}{(10)(0.04)} \ln \left( \frac{400 - 300}{400 - 350} \right)$$

T = 166.32 s

**Example:-**. A solid sphere of copper of radius R and a hollow sphere the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster?

**Solution:** Net rate of heat radiation  $\left(\frac{dQ}{dt}\right)$  will be same in both the cases, as temperature and area are same.

Therefore, from equation

$$\operatorname{ms}\left(-\frac{\mathrm{d}\theta}{\mathrm{d}t}\right) = \frac{\mathrm{d}Q}{\mathrm{d}t} \quad \text{or} \quad -\frac{\mathrm{d}\theta}{\mathrm{d}t} \propto \frac{1}{\mathrm{m}}$$

The hollow sphere will cool faster as its mass is less.

*Example:*-. A room is maintained at 20°C by a heater of resistance 20Ω connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area  $1 \text{m}^2$  and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal m<sup>-1</sup> s<sup>-1</sup> (°C)<sup>-1</sup> and mechanical equivalent of heat is 4.2 J cal<sup>-1</sup>

**Solution:** Power produced by heater = rate of heat flow through window

Substituting the value we have

$$\theta = 20 - \frac{(200)^2(0.2 \times 10^{-2})}{(0.2 \times 4.2)(1)(20)} = 15.24^{\circ}\text{C}$$

## Day 2 and Day 3 Main & Advance Level Problem

Please watch videos for the questions and also practice online assignments