Chapter 3

Wave Optics

Day – 1

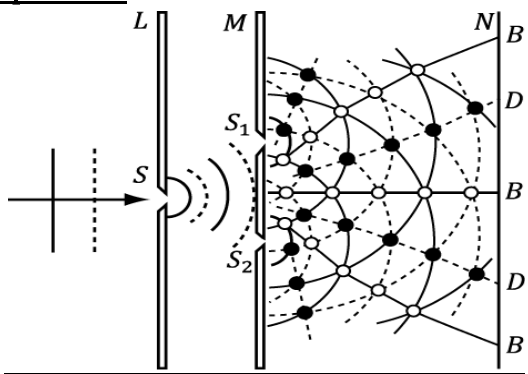
Principle of Superposition

When two, or more, waves travel simultaneously in a medium, the resultant displacement at each point of the medium at any instant is equal to the vector sum of the displacements produced by the two waves separately.

Interference of Light Waves

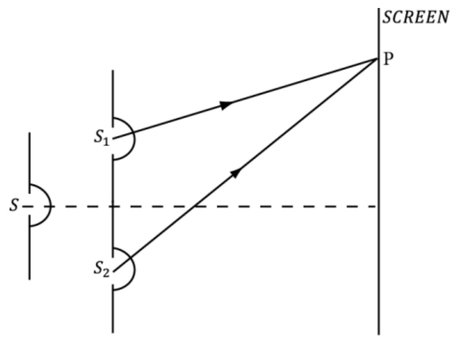
The re-distribution of light intensity due to the superposition to two light wave is called 'interference of light'.

Young' Double Slit Experiment



Conditions for Constructive and Destructive Interference (Maxima and Minima)

Let S be a narrow slit illuminated by a monochromatic source of light, and S_1 and S_2 two similar parallel slits very close together and equidistant from S. suppose the waves from S reach S_1 and S_2 in the same phase. Then, beyond S_1 and S_2, the waves proceed as if they started from S_1 and S_2. Let us find the resultant intensity of light at point P on a screen placed parallel to S_1 and S_2 Let a_1 and a_2 be the amplitudes of the light waves from S_1 and S_2 respectively. The waves arrive at P, having traversed different paths S_1P and S_2P. Hence they are superposed at P with a phase



difference ϕ, given by

$$\phi = \frac{2\pi}{\lambda}x$$

Where x is the path difference (S_2P-S_1P) between the waves at P and λ is the wavelength of light The displacement at P due to the simple harmonic waves from S1 and S_2 may then be represented by

$$y_1 = a_1 \sin \omega t \ldots\ldots (i)$$

And

$$y_2 = a_2 \sin(\omega t + \phi) \ldots\ldots (ii)$$

$\omega/2\pi$ is the frequency of each wave. By the principle of superposition, the resultant displacement at point P is given by

$$y = y_1 + y_2$$
$$= a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$
$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$
$$= \sin \omega t(a_1 + a_2 \cos \phi) + \cos \omega t(a_2 \sin \phi)$$

Let

$$a_1 + a_2 \cos \phi = R \cos \theta \ \ldots\ldots (iii)$$

And

$$a_2 \sin \phi = R \sin \theta \ldots\ldots. (iv)$$

Where R and θ are new constants. Then

$$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

Or

$$y = R \sin(\omega t + \theta)$$

This equation is similar to eq. (i) and (ii). Hence the resultant displacement at point P is due to a similar wave of amplitude R. To determine R, we square eq. (iii) and (iv) and then add

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2$$

Or

$$R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

The intensity is directly proportional to the square of the amplitude. Hence the resultant intensity I at point P is given by

$$I = KR^2 = Ka_1^2 + Ka_2^2 + K2a_1a_2 \cos \phi$$

Where K is constant of proportionality. If I_1 and I_2 be the separate intensities of the two interfering waves, then

$$I_1 = Ka_1^2 \quad and \ \ I_2 = Ka_2^2$$
$$I = I_1 + I_2\sqrt{I_1I_2} \cos \phi \ \ldots \ \ldots. (v)$$

Thus, the resultant intensity at any point depends upon the phase difference ϕ between the two waves at that point.

Constructive Interference

It follows from eq. (v) that the intensity of light at point P is maximum, if cosϕ = +1, that is

$$\phi = 0, 2\pi, 4\pi, \dots \dots$$

Or

$$\phi = 2m\pi,$$
$$where\ m = 0,1,2 \dots\ \dots$$

Then from eq. (v), we have

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = K(a_1 + a_2)^2 \dots \dots (vi)$$

Thus, at points where the two interfering waves meet in the same phase (that is, the phase difference between them is 0, 2π, 4π,…) the resultant intensity is maximum. Its value $K(a_1 + a_2)^2$ is greater than the sum of the intensities $K(a_1^2 + a_2^2)$ of the two waves. At these points, the resultant amplitude R is equal to the sum of the amplitudes $(a_1 + a_2)$ of the two waves. This is constructive interference

We can find the condition for constructive interference in terms of path difference x between the interfering waves also. We know that

$$x = \frac{\lambda}{2\pi}\phi$$

For maximum intensity, ϕ = 2m π, therefore

$$x = \frac{\lambda}{2\pi}(2m\pi)$$

Or

$$x = m\lambda,$$
$$m = 0,1,2, \dots..$$

Thus, **the intensity of light at a point is maximum if the path difference between the interfering waves at that point is zero or an integral multiple of wavelength λ** (that is, 0, λ, 2λ, 3λ, ……)

Destructive Interference

It follows from eq. (v) that the intensity of light at point P is minimum, if cosϕ = -1, that is

$$\phi = \pi, 3\pi, 5\pi, \dots \dots.$$

Or

$$\phi = (2m - 1)\pi, where\ m = 1,2,3, \dots$$

Then from eq. (v), we have

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$= \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2 = K(a_1 \sim a_2)^2, \dots \dots (vii)$$

Thus, **at points where the two interfering waves meet in opposite phase (that is, the phase difference between them is π, 3π, 5π,…)the resultant intensity is minimum.** Its value $K(a_1 \sim a_2)^2$ is less than the sum of the intensities of the two waves. At these points the resultant amplitude R is equal to the difference of the amplitudes $(a_1 \sim a_2)$ of the two waves. If the amplitudes of the two waves be equal $(a_1 = a_2)$ the resultant intensity at these points becomes zero. This is destructive interference.

In terms of path difference x, where x = λ/2π ϕ, we have for minimum intensity

$$x = \frac{\lambda}{2\pi}(2m-1)\pi$$

Or

$$x = (2m-1)\frac{\lambda}{2} \quad where\ m = 1,2,3, \ldots$$

Thus, **the intensity of light at a point is minimum if the path difference between the interfering waves at that point is an odd integral multiple of half- wavelength λ/2** (that is, λ/2, 3λ/2, 5λ/2,……)

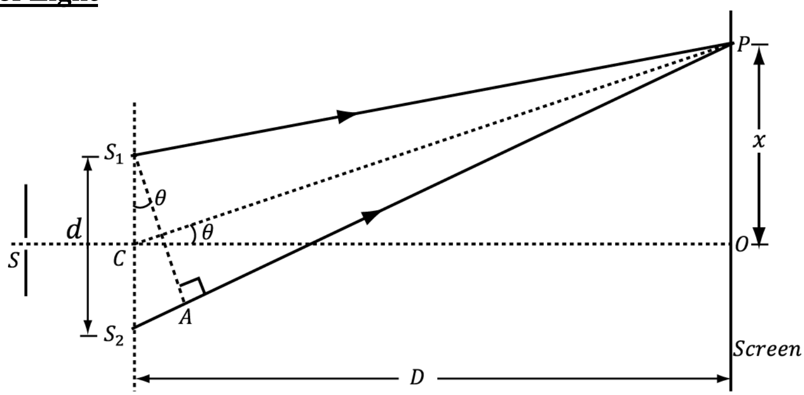
As we move on the screen, the path difference between the two waves gradually changes and there is a variation in the intensity of light, being alternately maximum and minimum. This is called the 'interference pattern'

From eq.(vi) and (vii), we have

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 \sim a_1)^2}$$

This is the ratio of the intensities of light at maxima and minima, that is, the ratio of intensities of bright and dark fringes

Position of Maxima and Minima in Interference Pattern: Fringe-width: Determination of Wavelength of Light



Let a bright fringe be formed at the point P on the screen. Let the right-bisector CO of the double-slit S_1S_2 meet the screen at the point P from O be x. the wavelets starting from $S_1\ and\ S_2$ traverse different distances $S_1P\ and\ S_2P$ to reach P. thus , the path difference between them is $(S_2P - S_1P)$.Let S_1 A be a perpendicular drawn on S_2P from S_1 . Then, the path difference between the two waves at the point P is

$$(S_2P - S_1P) = S_2A$$

The triangles S_1S_2A and PCO are similar. Therefore

$$\frac{S_2A}{S_1S_2} = \frac{OP}{CP}$$

The distance CO is large in comparison to S_1S_2 and so CP may be taken equal to CO

$$\frac{S_2A}{S_1S_2} = \frac{OP}{CO}$$

Or

$$\frac{S_2A}{d} = \frac{xd}{d}$$

Positions of Bright Fringes

The intensity of light will be maximum at those place where the path difference between the interfering light-waves is zero or an integral multiple of $\lambda(0, \lambda, 2\lambda, \dots \dots)$.Hence for maximum intensity (bright fringe), we have

$$\frac{xd}{D} = m\lambda \ where\ m = 0,1,2, \dots$$

Or

$$x = m\frac{D\lambda}{d} \dots (i)$$

Positions of Dark Fringes

The intensity of light will be minimum at those places where the path difference between the interfering light-waves is an odd integral multiple of $\lambda/2(\lambda/2, 3\lambda/2, 5\lambda/2, ..)$

Hence for minimum intensity (dark fringe), we have

$$\frac{xd}{D} = (2m-1)\frac{\lambda}{2} = \left(m - \frac{1}{2}\right)\lambda, where\ m = 1,2,3, \dots$$

Or

$$x = \left(m - \frac{1}{2}\right)\frac{D\lambda}{d} \dots (ii)$$

Putting m = 1 in this formula we get the position of the first dark fringe, putting m =2 the position of the second dark fringe … … and so on

A comparison of eq. (ii) with eq.(i) show that dark fringes are situated in between bright fringes.

Fringe – Width

The distance between two consecutive bright or dark fringes is called ‘fringe- width’ let $x_m\ and\ x_{m+1}$ be the distances of the m th and the $(m+1)$ th bright fringes respectively from the central fringe O. then

$$x_m = m\frac{D\lambda}{d}$$

And

$$x_{m+1} = (m+1)\frac{D\lambda}{d}$$

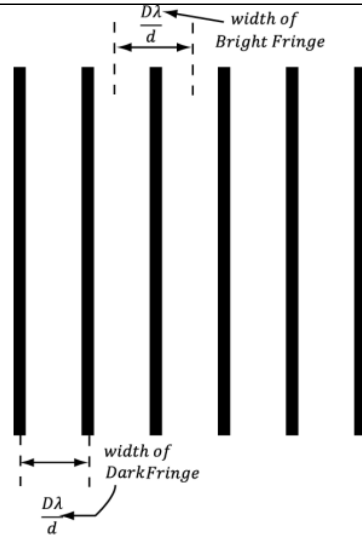
Therefore, the distance between the m th and the $(m+1)$th bright fringes, that is, the fringe – width is given by

$$x_{m+1} - x_m = (m+1)\frac{D\lambda}{d} - m\frac{D\lambda}{d} = \frac{D\lambda}{d}$$

We see that the fringe-width is free from m, that is **the width of all bright fringes is the same** (provided the distance D of the screen from the slits is much larger than the separation d between the slits). Similarly, it can be shown that **the width of all dark fringes is also the same** and is $D\lambda/d$. The fringe-width is denoted by W. Thus

$$W = \frac{D\lambda}{d} \dots (iii)$$

Measuring fringe-width W, the wavelength of light λ can be calculated



Angular Fringe-Width

The angular position of the m th bright fringe is

$$\theta_m = \frac{x_m}{D} = \frac{\left(mD\frac{\lambda}{d}\right)}{D} = \frac{m\lambda}{d}$$

And that of the $(m+1)th$ bright fringe is

$$\theta_{m+1} - \theta_m = \frac{(m+1)\lambda}{d} - \frac{m\lambda}{d}$$

Or

$$\theta = \frac{\lambda}{d}$$

Illustration

In Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance of the fourth bright fringe from the central bright fringe is found to be 1.2 cm. Compute the wavelength of light.

Solution

The position of the m th bright fringe with respect to the central (zero-order) fringe is given by

$$x = m\frac{D\lambda}{d}$$

Where D is distance of the screen from the slits having separation d

$$\therefore\ \lambda = \frac{xd}{mD}$$

Here $x = 1.2\ cm = 1.2 \times 10^{-2} m, d = 0.28\ mm = 0.28 \times 10^{-3} m, m = 4 and\ D = 1.4\ m$

$$\therefore\ \lambda = \frac{(1.2 \times 10^{-2}) \times (0.28 \times 10^{-3})}{4 \times 1.4}$$

$$= 6 \times 10^{-7} k\ m = 6000\dot{A}$$

Illustration

In Young's double-slit experiment, using light of wavelength 400 nm, interference fringes of width X are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe-width on the screen to be the same in the two cases, find the ratio of distances between the screen and the plane of the interfering sources in the two arrangements

Solution

The fringe-width is given by

$$W = \frac{D\lambda}{d}$$

For the first setting, we have

$$X = \frac{D \times 400\ nm}{d} \dots \dots (i)$$

In the second setting, separation between the slits is reduced to $d/2$, and the wavelength of light is increased to 600 nm, Then, to observe the same fringe-width X, the distance between the screen and the sources is D` (say). Then

$$X = \frac{D' \times 600\ nm}{d/2} \dots \dots (ii)$$

Equating (i) and (ii), we have

$$\frac{D \times 400\ nm}{d} = \frac{D' \times 600\ nm}{d/2}$$

$$= \frac{D' \times 600\ nm \times 2}{d}$$

Or

$$\frac{D}{D'} = \frac{600\ nm \times 2}{400\ nm} = 3:1$$

Illustration

In a Young's double-slit arrangement, a source of wavelength Å is used. The screen is placed 1 m from the slits. Fringes, formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes if they subtend an angle more than 1minute of arc. Calculate the maximum distance between the slits so that the fringes are clearly visible. Using this information, calculate the positions of 3 rd bright and 5 th dark fringes from the centre of the screen

Solution

The angular distance between two neighbouring fringes is given by

$$\theta = \frac{\lambda}{d}$$

Where d is the distance the two slits in Young's arrangement

For the student $\theta_{min} = 1\ minute$ of arc

$$= \left(\frac{1}{60}\right)^{0} = \frac{1}{60} \times \frac{\pi}{180} radian$$

$$\therefore d_{max} = \frac{\lambda}{\theta_{min}} = \frac{6000 \times 10^{-10} m}{\pi/(60 \times 180)}$$

$$= 2.064 \times 10^{-3} m = 2.064\ mm$$

From this, we have

$$\frac{D\lambda}{d_{max}} = \frac{1m \times (6000 \times 10^{-10} m)}{2.064 \times 10^{-3} m} = 2.91 \times 10^{-4} m$$

The position of a bright fringe with respect to the central bright fringe on the screen is given by

$$x_m = m\frac{D\lambda}{d}\ m = 0,1,2,3, \dots$$

For the third bright fringe, m = 3

$$\therefore x_3 = 3 \times (2.91 \times 10^{-4} m) = 8.73 \times 10^{-4} m = 0.873$$

The position of a dark fringe is given by

$$x_m = \left(m - \frac{1}{2}\right)\frac{D\lambda}{d}, \qquad m = 1,2,3, \dots \dots$$

For the fifth dark fringe, $m = 5$

$$\therefore x_5 = \frac{9}{2} \times (2.91 \times 10^{-4} m) = 13.1 \times 10^{-4} = 1.31\ mm$$

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