Chapter 2

Geometrical Optics (Refraction)

Day – 1

Refraction

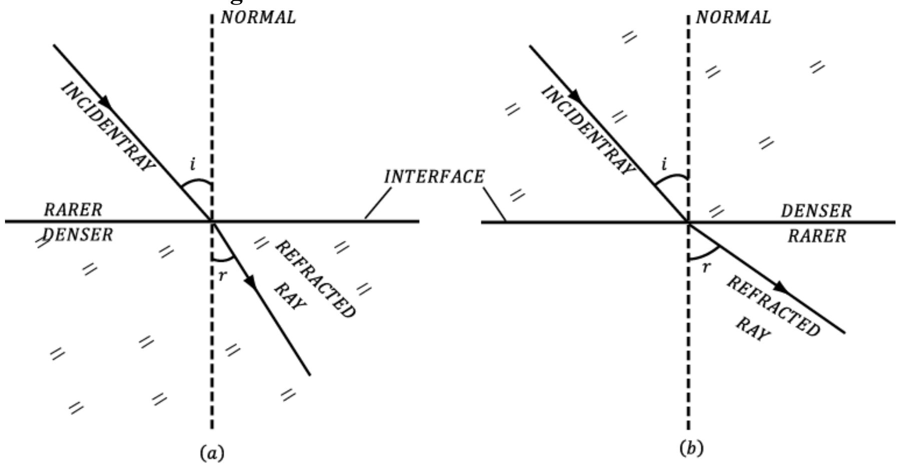
Refraction at Plane Surfaces

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

Where v_1 and v_2 are the speeds of light in media 1 and 2 respectively and μ_1 and μ_2 are the refractive indices of media 1and 2 respectively. For vacuum, the refractive index μ equals 1. For air also, it is very close to 1.

Refraction of Light

Bending of the light – ray from its path in passing from one medium to the other medium Is called ‘refraction’ of light.



(i) The incident ray, the refracted ray and the normal to the interface at the incident point all lie in the same plane.

(ii) For any two media and for light of a given colour (wavelength), the ratio of the sine of the angle of incidence to the sin of the angle of refraction is a constant.

$$\frac{\sin i}{\sin r} = \text{constant.}$$

$$\frac{\sin i}{\sin r} = \ {}_1n_2$$

$$\frac{\sin i}{\sin r} = \ {}_an_g$$

Cause of Refraction

Refraction of light occurs because the speed of light is different in different media

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

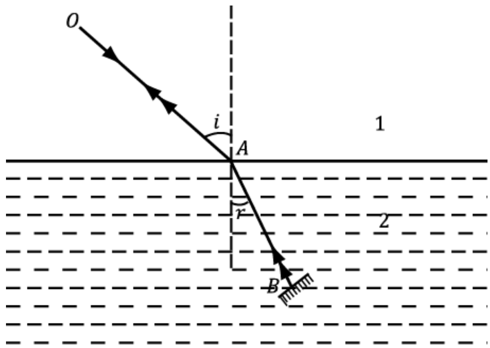
$$\therefore\ {}_1n_2 = \frac{v_1}{v_2} = \frac{\text{speed of light in the first medium}}{\text{speed of light in the second medium}}$$

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$$

In the process of refraction; the speed, wavelength and the intensity of light change; while the frequency of light wave remains unchanged.

Reversibility of Light

When a light- ray after suffering any number of reflections and refractions, has it final path reversed, it travels back along its entire initial path.



$${}_1n_2 = \frac{\sin i}{\sin r} \ldots\ldots (i)$$

$${}_2n_1 = \frac{\sin r}{\sin i} \ldots\ldots (ii)$$

$${}_1n_2 \times {}_2n_1 = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

$$\mathbf{{}_1n_2 = \frac{1}{{}_2n_1}}$$

The refractive index of medium 2 relative to medium 1 Is equal to the reciprocal of the refractive index of medium 1 relative to medium 2.

Refraction through Parallel Multiple Media

If i_1 and r_1 be the angles of incidence and refraction respectively at A, the interface of media 1 and 2, then, by Snell’s law, the have

$${}_1n_2 = \frac{\sin i_1}{\sin r_1} \ldots\ldots (i)$$

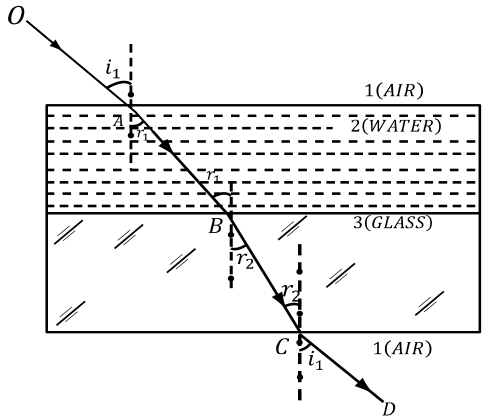
$${}_2n_3 = \frac{\sin r_1}{\sin r_2} \ldots\ldots (ii)$$

$${}_3n_1 = \frac{\sin r_2}{\sin i_1} \ldots\ldots (iii)$$

$$_1n_2 \times\ _2n_3 \times\ _3n_1$$

$$= \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_1}$$

$$= 1$$

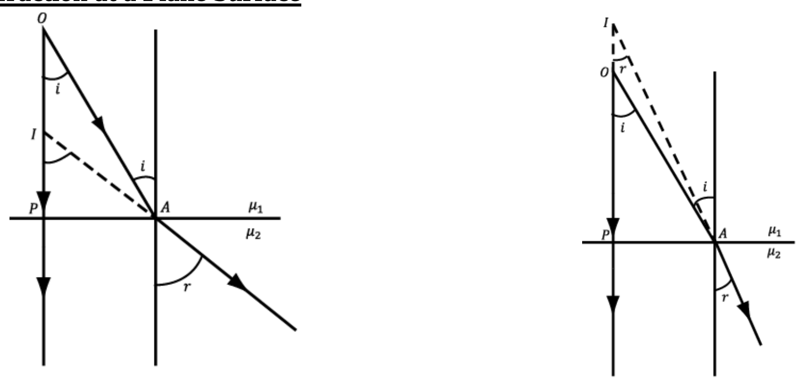


$$_1\mathrm{n}_2 \times\ _2\mathrm{n}_3 = \frac{1}{_3\mathrm{n}_1}$$

$$_1\mathrm{n}_3 =\ _1\mathrm{n}_2 \times\ _2\mathrm{n}_3$$

$$_2\mathrm{n}_3 = \frac{_1\mathrm{n}_3}{_1\mathrm{n}_2}$$

Image due to Refraction at a Plane Surface



(a) (b)

$$\sin i \approx \tan i = \frac{\mathrm{PA}}{\mathrm{PO}}$$

$$\sin r \approx \tan r = \frac{\mathrm{PA}}{\mathrm{PI}}$$

$$\text{Thus } \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

$$\left(\frac{\mathrm{PA}}{\mathrm{PO}}\right) \cdot \left(\frac{\mathrm{PI}}{\mathrm{PA}}\right) = \frac{\mathrm{PI}}{\mathrm{PO}} \qquad \text{... (i)}$$

Suppose medium 2 is air and observer looks at the image from this medium. The real depth of the object inside medium 1is PO whereas the depth as it appears to the observer is PI. Writing $\mu_2=1$ and $\mu_1=\mu$, equation (i) gives

$$\frac{1}{\mu} = \frac{\text{Apparent depth}}{\text{Real depth}}$$

$$\Rightarrow \mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

The time image shifts closer to eye by an amount OI=PO-PI

$$\Rightarrow \left(\frac{\text{PO}-\text{PI}}{\text{PO}}\right) \text{PO}$$

$$\Rightarrow \left(1 - \frac{\text{PI}}{\text{PO}}\right) \text{PO}$$

$$\Rightarrow \Delta t = \left(1 - \frac{1}{\mu}\right) t.$$

Some Important Points

(i) The value of absolute refractive index μ is always greater or equal to one.

(ii) The value of refractive index depends upon material of medium, colour of light and temperature of medium.

(iii) When temperature increases, refractive index decrease.

(iv) Optical path is defined as product of geometrical path and refractive index.

$$\text{Optical path} = \mu x$$

(v) For a given time, optical path remains constant.

$$\text{i.e.}, \mu_1 x_1 = \mu_2 x_2 = \cdots \text{constant}$$

$$\therefore \; \mu_1 \frac{dx_1}{dt} = \mu_2 \frac{dx_2}{dt}$$

$$\therefore \mu_1 c_1 = \mu_2 c_2$$

$$\therefore \frac{\mu_2}{\mu_1} = \frac{c_1}{c_2}$$

$$\text{i.e.}, \mu \propto \frac{1}{c}$$

(vi) The frequency of light does not depend upon medium.

$$\therefore c_1 = f\lambda_1, c_2 = f\lambda_2$$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{c_1}{c_2} = \frac{\lambda_1}{\lambda_2}$$

$$\therefore \mu \propto \frac{1}{\lambda}$$

2. (a) When observer is in rarer medium and object is in denser medium.

$$\mu = \frac{\text{Real deapth}}{\text{Apparent depth}}$$

(b) When object is in air and observer is in denser medium.

$$\mu = \frac{\text{Real deapth}}{\text{Apparent depth}}$$

(c) The shift of object due to slab is

$$x = t\left(1 - \frac{1}{\mu}\right)$$

(i) This formula is only applicable when observer is in rarer medium.

(ii) The object shiftness does not depend upon the position of object.

(iii) Object shiftness takes place in the direction of incident ray.

(d) The equivalent refractive index of a combination of a number of slabs for normal incidence is

$$\mu = \frac{\sum t_i}{\sum \frac{t_i}{\mu_i}},$$

Here,

$$\sum t_i = t_1 + t_2 + \cdots \ldots.. \text{ and } \sum \frac{t_i}{\mu_i} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \cdots$$

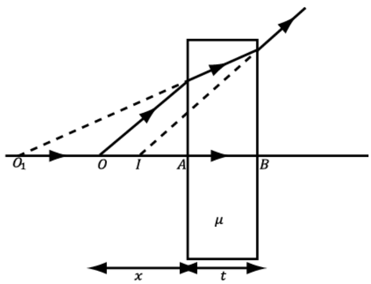
(e) The apparent depth due to a number of media is $\sum t_i/\mu_i$

Illustration

A point *O* is front of a transparent slab at a distance *x* from its closer surface. It is seen from the other side of the slab by light incident nearly normally to the slab. The thickness of the slab is t and its refractive index is μ. Show that the apparent shift in the position of the object is independent of *x* and find its value.

Solution

The situation is shown in figure. Because of the refraction at the first surface, the image of *O* is formed at O_1. For this refraction, the real depth is $AO = x$ and the apparent depth is AO_1. Also, the first medium is air and the second is the slab.



Thus,

$$\frac{x}{AO_1} = \frac{1}{\mu} \; or \; , AO_1 = \mu x$$

The point O_1 acts as the object for the refraction at the second surface. Due to this refraction, the image

of O_1 is formed at I. Thus,

$$\frac{BO_1}{BI} = \mu$$

$$\frac{AB + AO_1}{BI} = \mu$$

$$\Rightarrow \frac{t + \mu x}{BI} = \mu$$

$$\Rightarrow BI = x + \frac{t}{\mu}$$

The net shift is

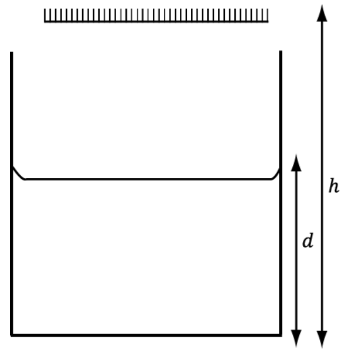
$$OI = OB - BI$$

$$\Rightarrow (x + t) - \left(x + \frac{t}{\mu}\right)$$

$$\Rightarrow t\left(1 - \frac{1}{\mu}\right), \text{which is independent of } x.$$

Illustration

Consider the situation shown in figure. A plane mirror is fixed at a height h above the bottom of a beaker containing water (refractive index μ) up to a height d. Find the position of the image of the bottom formed by the mirror.



Solution

The bottom of the beaker appears to be shifted up by a distance

$$\Delta t = \left(1 - \frac{1}{\mu}\right) d.$$

Thus, the apparent distance of the bottom from the mirror is

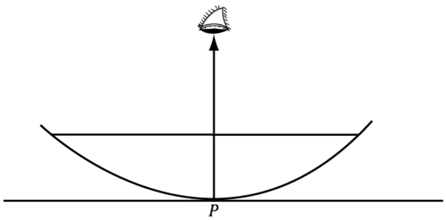
$$h - \Delta t = h - \left(1 - \frac{1}{\mu}\right) d = h - d + \frac{d}{\mu}.$$

The image formed behind the mirror at a distance

$$h - d + \frac{d}{\mu}.$$

Illustration

A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00cm. A small dust particle floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is1.33.

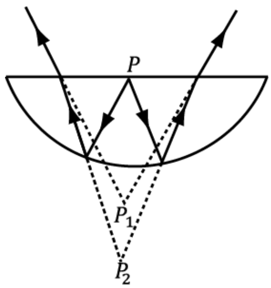


Solution

The ray diagram is shown in figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then R = -4cm. The object distance is u = -5cm. Using the mirror equation,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$
$$\Rightarrow \frac{1}{v} = \frac{2}{R} - \frac{1}{u}$$

$$\Rightarrow \frac{2}{-40\ cm} - \frac{1}{-5\ cm}$$
$$\Rightarrow \frac{6}{40cm}$$
$$v = 6.67\ cm$$



The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P1from the surface is 6.67 cm+5.00cm = 11.67 cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance

$$(11.67cm)\left(1 - \frac{1}{1.33}\right) = 2.92\ cm.$$

Thus, the final image is formed at a point (11.67-2.92) cm=8.75 cm below the water surface.

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