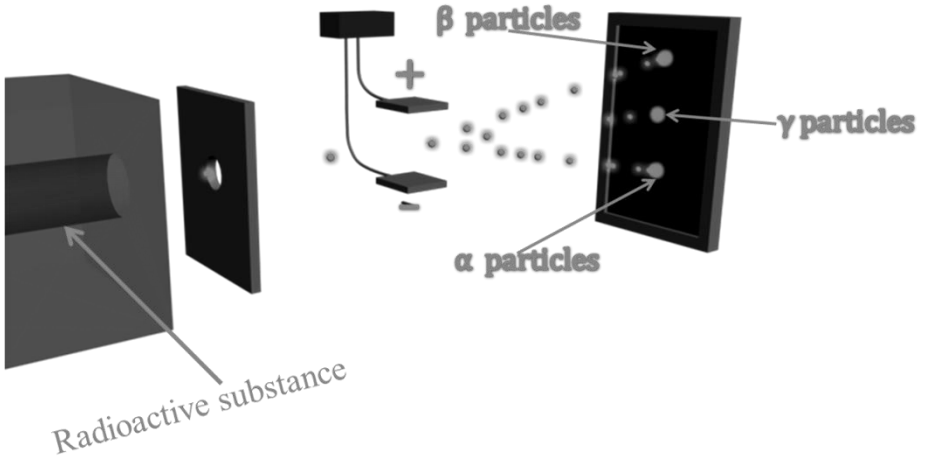
Chapter 2 Nuclear Physics

Day – 1

1. Radioactivity



1.1 α – decay

$$^{A}_{Z}X = {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Parent nucleus — Daughter nucleus

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$
$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

1.2 Radioactive Series

	Mass number	Parent	Half life	Stable product
1. Thorium	$4n$	$^{232}_{90}Th$	1.39×10^{10} years	$^{208}_{82}Pb$
2. Neptunium	$4n + 1$	$^{237}_{93}Np$	2.25×10^{6} years	$^{209}_{83}Bi$
3. Uranium	$4n + 2$	$^{238}_{92}U$	4.47×10^{9} years	$^{206}_{82}Pb$
4. Actinium	$4n + 3$	$^{235}_{92}U$	7.07×10^{8} years	$^{207}_{82}Pb$

Note:- Neptunium is now not found because $\left(t_{\frac{1}{2}}\right)_{Np} < \left(t_{\frac{1}{2}}\right)_{earth}$.

1.3 Nuclear Stability and Radioactivity

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by emitting particles and electromagnetic radiation, a process called ***radioactivity.*** The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The stable nuclides are shown by dots on the graph in figure, where the neutron number N and proton number (or atomic number) Z for each nuclide are plotted. Such a chart is called a Segre chart, after its inventor, the Italian-American physicist Emilio Segre (1905-1989).

Each blue line perpendicular to the line N = Z represents a specific value of the mass number A = Z + N. Most lines of constant A pass through only one or two stable nuclides; that is, there is usually a very narrow range of stability for a given mass number. The lines at A = 20, A = 40, A = 60, and A = 80 are examples. In four cases these lines pass through three stable nuclides, namely, at A = 96, 124, 130, and 136.

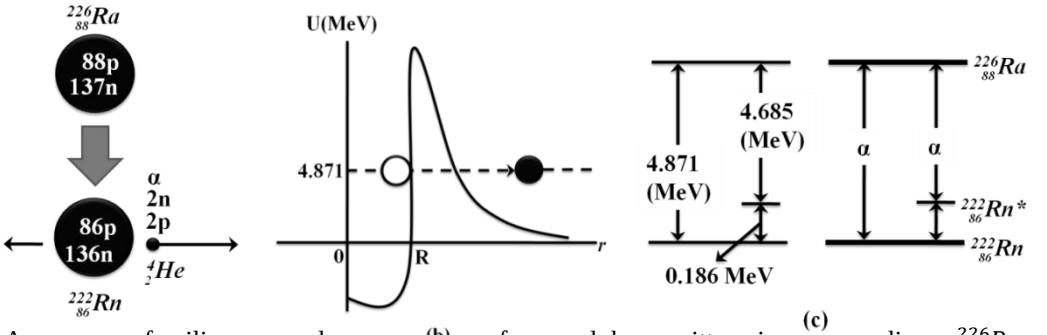
Our four stable nuclides have both odd Z and odd N:

$^{2}_{1}\mathrm{H},\ ^{6}_{3}\mathrm{Li},\ ^{10}_{5}\mathrm{B},\ ^{14}_{7}\mathrm{N}.$

These are called odd-odd nuclides. The absence of other odd-odd nuclides shows the influence of pairing. Also, there is no stable nuclide with A = 5 or A = 8.

1.4 Alpha Decay

Nearly 90% of the 2500 known nuclides are radioactive; they are not stable but decay into other nuclides. When unstable nuclides decay into different nuclides, they usually emit alpha (α) or beta (β) particles. An ***alpha particle*** is a ^{4}He nucleus, two protons and two neutrons bound together, with total spin zero. Alpha emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an alpha particle, its N and Z values each decrease by two and A decreases by four, moving it closer to stable territory on the Segre chart.



A familiar example of an alpha emitter is radium, $^{226}_{88}Ra$. The speed of the emitted alpha particle, determined from the curvature of its path in a transverse magnetic field, is about 1.5210^{7} m/s. This speed, although large, is only 5% of the speed of light, so we can use the nonrelativistic kinetic-energy expression $\mathrm{K} = \frac{1}{2}\mathrm{mv}^2$:

$$\mathrm{K} = \tfrac{1}{2}(6.64 \times 10^{-27}\ \mathrm{kg})(1.52 \times 10^{7}\ \mathrm{m/s})^2 = 7.7 \times 10^{-13}\ \mathrm{J} = 4.8\ \mathrm{MeV}.$$

You can use conservation of mass-energy to show that the alpha decay is possible whenever the mass of the original neutral atom is greater than sum of the masses of the final neutral atom and the neutral helium-4 atom. In alpha decay, the α particle tunnels through a potential-energy barrier, as shown in figure.You may want to review the discussion of tunneling.

1.5 Beta Decay

There are three different simple types of beta decay: beta-minus, beta-plus, and electron capture. A ***beta- minus particle*** (β^-) is an electron. It's not obvious how a nucleus can emit an electron if there aren't any electrons in the nucleus. Emission of a β^- involves transformation of a neutron into a proton, an electron, and a third particle called an antineutrino. In fact, if you freed a neutron from a nucleus, it would decay into a proton, an electron, and an antineutrino in an average time of about 15 minutes.

Beta particle can be indentified and their speeds can be measured with techniques that are similar to the Thomson experiments we described. The speeds of beta particles range up to 0.9995 of the speed of light, so their motion is highly relativistic. They are emitted with a continuous spectrum of energies.

This third particle is an antineutrino, the antiparticle of a ***neutrino***. The symbol for a neutrino is ν (the Greek letter "nu"). Both the neutrino and the antineutrino have zero charge and zero (or very small) mass and therefore produce very little observable effect when passing through matter. Both evaded detection until 1953, when Frederick Reines and Cly Cowan succeeded in observing the antineutrino directly. We now know that there are at least three varieties of neutrinos, each with its corresponding antineutrino; one is associated with beta decay and the other two are associated with the decay of two unstable particles, the *mu*on and the tau particle. The antineutrino that is emitted in β^- decay is denoted as

$\bar{\nu}_e$. The basic process of β^- decay is

$$n \rightarrow p + \beta + \bar{\nu}_e.$$

Beta minus decay usually occurs with nuclides for which the neutron-to-proton ratio N/Z is too large for stability. In beta decay, N decreases by one, Z increases by one and A doesn't change. you can use conservation of mass-energy to show that beta-minus decay can occur whenever the neutral atomic mass of the original atom is larger than that of the final atom. We have noted that beta decay occurs with nuclides that have too large a neutron-to-proton ratio N/Z. Nuclides for which N/Z is too small for stability can emit a positron, the electron's antiparticle, which is identical to the electron but with positive charge. The basic process, called beta-plus decay (β^+), is

$$p \rightarrow n + \beta^+ + \nu_e,$$

Where β^+ is a positron and ν_e is the electron neutrino. Beta-plus decay can occur whenever the neutral atomic mass of the original atom is at least two electron masses larger than that of the final atom; you can show this using conservation of mass-energy.

The third type of beta decay is electron capture. There are a few nuclides for which β^+ emission is not energetically possible but in which an orbital electron (usually in the K shell) can combine with a proton in the nucleus to form a neutron and neutrino. The neutron remains in the nucleus and the neutrino is emitted. The basic process is

$$p + \beta^- \rightarrow n + \nu_e.$$

You can use conservation of mass-energy to show that electron capture can occur whenever the neutral atomic mass of the original atom is larger than that of the final atom. In all types of beta decay, A remains constant.

1.6 Gamma Decay

The energy of internal motion of a nucleus is quantized. A typical nucleus has a set of allowed energy levels, including a ground state (state of lowest energy) and several excited states. Because of the great strength of nuclear interactions, excitation energies of nuclei are typically of the order of 1 MeV, compared with a few eV for atomic energy levels. In ordinary physicaland chemical transformations the nucleus always remains in its ground state. When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called gamma rays or gamma-ray photons, with typical energies of 10 keV to 5 MeV. This process is called gamma (γ) decay. For example: when a alpha particle with the smaller energy is emitted, the ^{222}Rn nucleus is left in an excited state. It then decays to its ground state by emitting a gamma-ray photon with energy

(4.871 – 4.685) MeV = 0.186 MeV.

1.7 Natural Radioactivity

Many radioactive elements occur in nature. For example, you are very slightly radioactive because of unstable nuclides such as carbon-14 and potassium-40 that are present throughout your body. The study of natural radioactivity began in 1896; one year after Rontgen discovered x rays. Henri Becquerel discovered a radiation from uranium salts that seemed similar to x rays. Intensive investigation in the following two decades by Marie and Pierre Curie, Ernest Rutherford, and many others revealed that the emissions consist of positively and negatively charged particles and neutral rays; they were given the names alpha, beta, and gamma because of their differing penetration characteristics.

The decaying nucleus is usually called the parent nucleus; the resulting nucleus is the daughter nucleus. When a radioactive nucleus decays, the daughter nucleus may also be unstable. In this case a series of successive decays occurs until a stable configuration is reached. Several such series are found in nature. The most abundant radioactive nuclide found on earth is the uranium isotope ^{238}U, which undergoes a series of 14 decays, including eight α emissions and six β^- emissions, terminating at a stable isotope of lead, ^{206}Pb.

The decays can also be represented in equation form; the first two decays in the series are written as

$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + \alpha$$
$$^{234}\mathrm{Th} \rightarrow {}^{234}\mathrm{Pa} + \beta^-,$$

or more briefly as

$$^{238}\mathrm{U} \xrightarrow{\alpha} {}^{234}\mathrm{Th},$$
$$^{234}\mathrm{Th} \xrightarrow{\beta^-} {}^{234}\mathrm{Pa}.$$

In the second process, the beta decay leaves the daughter nucleus 234Pa in an excited state, from which it decays to the ground state by emitting a gamma-ray photon. An excited state is denoted by an asterisk, so we can represent the γ emission as

$$^{234}\mathrm{Pa}^* \longrightarrow {}^{234}\mathrm{Pa} + \gamma$$

or

$$^{234}\mathrm{Pa}^* \xrightarrow{\gamma} {}^{234}\mathrm{Pa}.$$

Many other decay series are known. Two of these occur in nature, one starting with the uncommon isotope ^{235}U and ending with ^{207}Pb, the other starting with thorium (^{232}Th) and ending with ^{208}Pb.

1.8 Activities and Half-Lives

Let N(t) be the (very large) number of radioactive nuclei in a sample at time t, and let dN(t) be the (negative)change in that number during a short time interval dt. (We'll use N(t) to minimize confusion with the neutron number N.) The number of decays during the interval dt is –dN(t). The rate of charge of N(t) is the negative quantity dN(t)/dt; thus –dN(t)/dt is called the decay rate or the activity of the specimen. The larger the number of nuclei in the specimen, the more nuclei decay during any time interval. That is, the activity is directly proportional to N(t); it equals a constant λ multiplied by N(t):

$$-\frac{dN(t)}{dt} = \lambda n(t)$$

The constant λ is called the decay constant, and it has different values for different nuclides. A large value of λ corresponds to rapid decay; a small value corresponds to slower decay. Solving the above equation for λ shows us that λ is the ratio of the number of decays per time to the number of remaining radioactive nuclei; λ can then be interpreted as the probability per time that any individual nucleus will decay.

To find the exponential function:

$$N(t) = N_0 e^{-\lambda t} \qquad \text{(number of remaining nuclei).}$$

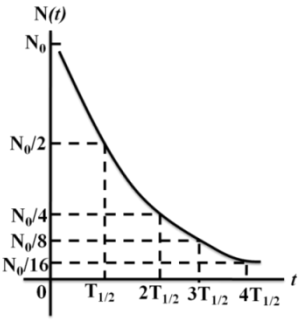


Figure is a graph of this function, showing the number of remaining nuclei N(t) as a function of time.

The half-life $T_{1/2}$ is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0. Then half of the remaining radioactive nuclei decay during a second interval $T_{1/2}$, and so on. The numbers remaining after successive half-lives are $N_0/2$, $N_0/4$, $N_0/8$, ……

To get the relation between the half-life $T_{1/2}$ and the decay constant λ, we set $N(t)/N_0 = 1/2$ and $t = T_{1/2}$ in the last equation, obtaining

$$\frac{1}{2} = e^{-\lambda T_{1/2}}.$$

We take logarithms of both sides and solve for $T_{1/2}$:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

The mean lifetime T_{mean}, generally called the lifetime, of a nucleus or unstable particle is proportional to the half-life $T_{1/2}$:

$$T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \quad \text{(lifetime } T_{mean}\text{, decay constant } \lambda\text{, and half-life } T_{1/2}\text{).}$$

In particle physics the life of an unstable particle is usually described by the lifetime, not the half-life.

Caution:- It is sometimes implied that any radioactive sample will be safe after a half-life has passed. That's wrong. If your radioactive waste initially has ten times too much activity for safety, it is not safe after one half-life, when it still has five times too much. Even after three half-lives it still

has 25% more activity than is safe. The number of radioactive nuclei and the activity approach zero only as t approaches infinity.

A common unit of activity is the curie, abbreviated Ci, which is defined to be 3.70×10^{10} decays per second. This is approximately equal to the activity of one gram of radium. The SI unit of activity is the Becquerel, abbreviated Bq. On Becquerel is one decay per second, so

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{Bq} = 3.70 \times 10^{10}\ \text{decays/s}.$$

1.9 β – decay

(i) β^- – decay

$$^{A}_{Z}\text{X} \xrightarrow{\beta^-} {}^{A}_{Z+1}\text{Y}$$

$$n \rightarrow \text{P} + e^- + \bar{\nu}$$

Example:- $^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$

(ii) β^+ – decay

$$^{A}_{Z}X \xrightarrow{\beta^+} {}_{Z-1}{}^{A}Y$$

$$P \rightarrow n + e^+ + \nu$$

Positron

Example:- $^{13}_{7}\text{C} \rightarrow {}^{13}_{6}\text{N} + \text{e}^+ + \nu$

1.10 γ – decay

$$^{A}_{Z}X^* \longrightarrow {}^{A}_{Z}X + \gamma$$

Example:- $^{12}_{5}B \rightarrow {}^{12}_{6}C^* + e^- + \bar{\nu}$

$$^{12}_{6}C^* \rightarrow {}^{12}_{6}C + \gamma$$

1.11 Decay Law

$$-\frac{\text{dN}}{\text{dt}} \propto \text{N}$$

$$-\frac{\text{dN}}{\text{dt}} = \lambda\text{N}$$

where λ is decay constant.

$$\int_{\text{N}_0}^{\text{N}} \frac{\text{dN}}{\text{N}} = -\lambda \int_0^{\text{t}} \text{dt}$$

$$\ln\frac{\text{N}}{\text{N}_0} = -\lambda\text{t} \quad \Rightarrow \quad \mathbf{N = N_0 e^{-\lambda t}}$$

1.12 Decayed nuclei

$$= \text{N}_0 - \text{N}$$

$$= \text{N}_0 - \text{N}_0\text{e}^{-\lambda\text{t}}$$

$$= \text{N}_0\left(1 - \text{e}^{-\lambda\text{t}}\right)$$

1.13 Probability for survival

$$\text{P}_{(\text{survival})} = \frac{N}{N_0} = e^{-\lambda t}$$

1.14 Half life

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = \frac{ln\,2}{\lambda} = \frac{0.693}{\lambda}$$

1.15 Mean life

$$t_{av} = \frac{\int_{N_0}^{N} t\, dN}{N_0}$$

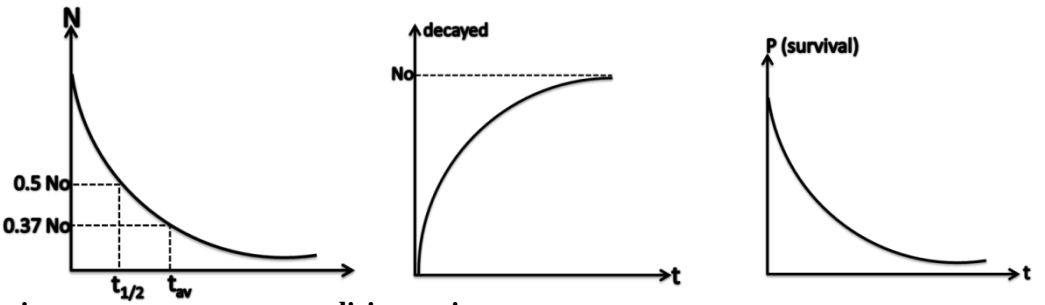
$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} \quad \Rightarrow \quad t_{av} = \frac{\int_0^{\infty} t N_0 A e^{-\lambda t}\, dt}{N_0}$$

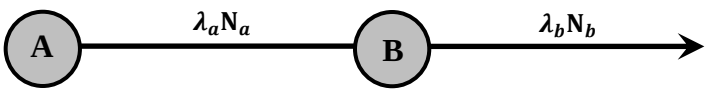
$$\mathbf{t_{av} = \frac{1}{\lambda}}$$

Also $\mathbf{t_{\frac{1}{2}} = 0.693\, t_{av}}$

or $\mathbf{t_{av} = 1.44\, t_{\frac{1}{2}}}$



1.16 Successive disinteration



At t = 0 N_0 Product 0

At t = t $N_a = N_0 e^{-\lambda_a t}$ N_b

$$\frac{dN_b}{dt} = \lambda_a N_a - \lambda_b N_b$$

$$= \lambda_a N_0 e^{-\lambda_a t} - \lambda_b N_b$$

$$dN_b + \lambda_b N_b dt = \lambda_a N_0 e^{-\lambda_a t}$$

$$e^{-\lambda_b t} dN_b + \lambda_b N_b e^{-\lambda_b t} dt = \lambda_a N_0 e^{-\lambda_b t} e^{-\lambda_a t}$$

Solving…

$$N_b = \frac{\lambda_a N_0}{\lambda_b - \lambda_a} \left(e^{-\lambda_a t} - e^{-\lambda_b t}\right)$$

Illustration

A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half life of 10.0 min (600 s). (a) How many nuclei are present initially? (b) What is the activity initially? (c) What is the activity after 1.00 h? (d) After approximately how long will the activity drop to less than one per second?

Solution

(a) Since the atomic mass is 13.0, then 13.0 g will contain 1.49×10^{23} nuclei (Avogadro's number). Since we have only 1.49×10^{-6} g, the number of nuclei, N_0, that we have initially is given by the ratio

$$\frac{N_0}{1.49\times10^{-6}} = \frac{6.02\times10^{23}}{13.0}$$

So $N_0 = 6.90 \times 10^{16}$ nuclei

(b) $\lambda = \frac{0.693}{600} = 1.16 \times 10^{-3}\ s^{-1}$

Then, at t = 0 s

$\left[\frac{dN}{dt}\right]_0 = \lambda N_0 = 1.16 \times 10^{-3} \times 6.90 \times 10^{16} = 8.00 \times 10^{13}\ s^{-1}$

(c) After 1.00 h = 3600 s, the activity will be

$$\frac{dN}{dt} = \left[\frac{dN}{dt}\right]_0 e^{-\lambda t}$$

$$= 8.00 \times 10^{13} \times e^{-(1.16\times10^{-3}\times3600)} = 1.23 \times 10^{12}\ s^{-1}$$

This result can be obtained in another way: since 1.00 h represents six half lives (6×10.0 min), the activity will decrease to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$ of the original value, or $\frac{8.00\times10^{13}\ s^{-1}}{64} = 1.25 \times 10^{12} s^{-1}$. (The slight discrepancy between the two values arises because we kept only three significant figures.)

(d) We want to determine the time *t* when dN/dt = 1.00 s^{-1}. We have

$$e^{-\lambda t} = \frac{\frac{dN}{dt}}{\left(\frac{dN}{dt}\right)_0} = \frac{1.00\ s^{-1}}{8.00\times10^{13}\ s^{-1}} = 1.25 \times 10^{-14}$$

We take the natural log (In) of both sides and divide by λ to find

$$t = \frac{-In(1.25\times10^{-14})}{\lambda} = \frac{32.0}{1.16\times10^{-3}\ s^{-1}} = 2.76 \times 10^4\ s$$

Illustration

A bone suspected to have originated during the period of Ashoka the Great, was found in Bihar. Accelerator techniques gave its $\frac{^{14}C}{^{12}C}$ ratio is 1.1×10^{-12}. Is the bone old enough to have belonged to that period ? (Take initial ratio of ^{14}C with $^{12}C = 1.2 \times 10^{-12}$)

Solution

The initial ratio of $\frac{^{14}C}{^{12}C}$ at the time of death was $R_0 = 1.2 \times 10^{-12}$.

Decay equation is $N(^{14}C) = N_0 e^{-\lambda t}$

The ratio of ions is given by

$$R = \frac{N(^{14}C)}{N(^{12}C)} = \frac{N_0(^{14}C)e^{-\lambda t}}{N(^{12}C)} = R_0 e^{-\lambda t}$$

where R_0 is the original ratio. We can solve this equation for t.

$$e^{-\lambda t} = \frac{R}{R_0}$$

$$t = \frac{In\left(\frac{R_0}{R}\right)}{\lambda} = \frac{In\left(\frac{R_0}{R}\right)}{In2} T_{1/2}$$

$$= \frac{0.087}{0.693} \times 5730 \text{ years}$$

$$= 719 \text{ years}$$

On substituting numerical values we get the age of the bone as 719 years. The bone does not date from Ashoka times, but from the Middle Age.

Illustration

A small quantity of a solution containing N^{24} radio nuclide (half life 15 hr) of activity 1.0 micro-curie is injected into the blood o a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegration per minute. Determine the total volume of blood in

the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.

Solution

Initial activity, $R_0 = 1\mu Ci = 3.7\times10^4$ decay/s

Final activity, $R_1 = \frac{296}{60} = 4.933$ decay/s

This final activity is in 1 cm^3 of the blood after 5 hours. Let R be the total final activity (in total volume of blood) after 5 hours. From law of radioactive decay, we have

$$R = R_0 e^{-\lambda t}$$

Now $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{15} = 0.0462$ /hr

T = 5 hours

$$R = (3.7 \times 10^4)e^{-0.0462\times5}$$
$$= (3.7 \times 10^4)e^{-0.231}$$
$$= (3.7 \times 10^4) \times (0.7937)$$
$$= 29367 \text{ decay/s}$$

$\therefore$ Total volume of blood $= \frac{\text{Total activity R}}{\text{Activity per cm}^3\ R_1}$

$$= \frac{29367}{4.933} \text{ cm}^3$$
$$= 5953.17 \text{ cm}^3$$
$$\simeq 5.953 \text{ litre}$$

1.17 Radioactive Dating

An interesting application of radioactivity is the dating of archeological and geological specimens by measuring to concentration of radioactive isotopes. The most familiar example is carbon dating. The unstable isotope ^{14}C, produced during nuclear reactions in the atmosphere that result from cosmic-ray bombardment, gives a small proportion of ^{14}C in the CO_2 in the atmosphere. Plants that obtain their carbon from this source contain the same proportion of ^{14}C as the atmosphere. When a plant dies, it stops taking in carbon, and its ^{14}C β^- decays to ^{14}N with a half-life of 5730 years. By measuring the proportion of ^{14}C in the remains, we can determine how long ago the organism died.

One difficulty with radiocarbon dating is that the ^{14}C concentration in the atmosphere changes over long time intervals. Corrections can be made on the basis of other data such as measurements of tree rings that show annual growth cycles. Similar radioactive techniques are used with other isotopes for dating geologic specimens. Some rocks, for example, contain the unstable potassium isotope ^{40}K, a beta emitter that decays to the stable nuclide ^{40}Ar with a half-life of 2.4×10^8 y. The age of the rock can be determined by comparing the concentrations of ^{40}K and ^{40}Ar.

Practice Question online