Chapter 3 — Magnetic Flux and Gauss's law of Magnetism

Day – 1

Introduction

We define the magnetic flux ϕ_B through a surface just as we defined electric flux in connection with Gauss's law. We can divide any surface into elements of area dA. For each element we determine $B_\perp$ the component of $\vec{B}$ normal to the surface. (Be careful not to confuse ϕ with ϕ_B.) In general, this component varies from point to point on the surface. We define the magnetic flux $d\phi_B$ through this area as

$$d\phi_B = B_\perp dA = B\cos\phi\, dA = \vec{B}.d\vec{A}$$

$$\phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B}.d\vec{A}$$

(magnetic flux through a surface)

Magnetic flux is a scalar quantity. In the special case in which is uniform over a plane surface with total area A. $B_\perp$ and ϕ are the same at all points on the surface and

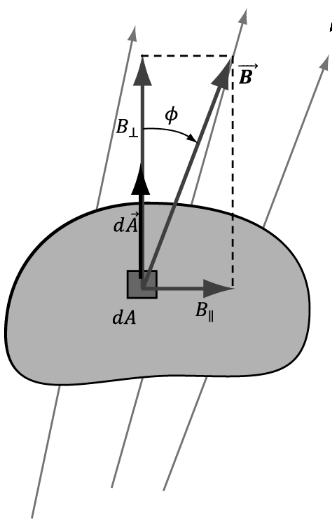
$$\phi_B = B_\perp A = BA\cos\phi$$

The SI unit of magnetic flux is equal to the unit of magnetic field (1T) times the unit of area (Im^2). This unit is called the Weber (1Wb), in honor of the German physicist Wilhelm Weber (1804 – 1891)

$$1Wb = 1T.m^2$$

$$1T = 1N/A.m$$

$$b = 1T.m^2 = 1N.m/A$$



The magnetic flux through an area element dA is defined to be $d\phi_B = B_\perp\, dA$.

In Gauss's law the total electric flux through a closed surface is proportional to the total electric charge enclosed by the surface. For example, if the closed surface encloses an electric dipole, the total electric flux is zero because the total charge is zero. The total magnetic flux through a closed surface would be proportional to the total magnetic charge enclosed. But we have mentioned that no magnetic monopole has ever been observed, despite intensive searches. We conclude that the total magnetic flux through a closed surface is always zero. Symbolically

$$\int \vec{B} = d\vec{A} = 0 \text{ (megnetic flux through any closed surface)}$$

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Caution

Unlike electric field lines that begin and end on electric charges, magnetic field lines never have end points; such a point would indicate the presence of a monopole. You might be tempted to draw magnetic field lines that begin at the north pole of a magnet and end at a south pole. But as Fig Shows, the field lines of a magnet actually continue through the interior of the magnet. Like all other magnetic field lines, they form closed loops.

For Gauss's law, which always deals with closed surface, the vector area element $d\vec{A}$ Eq. always points out of the surface. However, some applications of magnetic flux involve an open surface with a boundary line; there is then an ambiguity of sign in Eq. because of the two possible choices of direction for $d\vec{A}$. In these cases we choose one of the possible choices of direction for $d\vec{A}$. In these cases we choose one of the possible sides of the surface to be the "positive" side and use that choice consistently.

If the element of area dA in Eq. is at right angles to the field lines, then $B_{\perp} = B$; calling the area $dA_{\perp}$ we have

$$B = \frac{d\phi_B}{dA_{\perp}}$$

That is, the magnitude of magnetic field is equal to flux per unit area across an area at right angles to the magnetic field. For this reason, magnetic field $\vec{B}$ is sometimes called magnetic flux density

Induction Experiments

During the 1830s, several pioneering experiments with magnetically induced emf were carried out in England by Michael Faraday and in the United States by Joseph Henry (1797 – 1878), later the first director of the Smithsonian Institution. Fig shows several examples. In Fig a coil of wire is connected to a galvanometer. When the nearby magnet is stationary, the meter shows no current.

This isn't surprising; there is no source of emf in the circuit. But when we move the magnet either toward or away from the coil, the meter shows current in the circuit, but only while the magnet is moving. If we keep the magnet stationary and move the coil, we again detect a current during the motion. We call this an induced current, and the corresponding emf required to cause this current is called an induced emf.

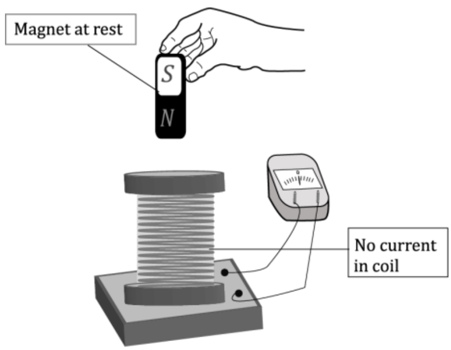
To explore further the common elements in these observations, let's consider a more detailed series of experiments with the situation shown in Fig. We connect a coil of wire to a galvanometer, then place the coil between the poles of an electromagnet whose magnetic field we can very. Here's what we observe

1. When there is no current in the electromagnet, so that $\vec{B} = 0$, the galvanometer shows no current.

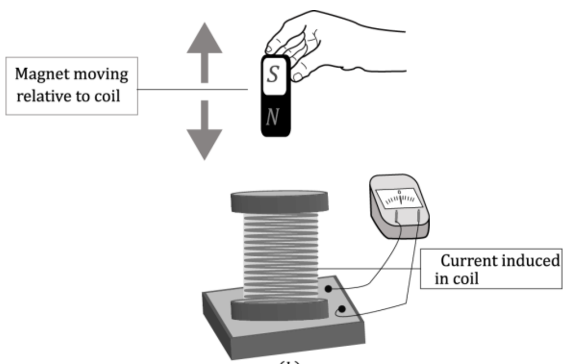
2. When the electromagnet is turned on, there is a momentary current through the meter $\vec{B}$ as increases

3. When $\vec{B}$ levels off at a steady value, the current drops to zero, no matter how large $\vec{B}$ is

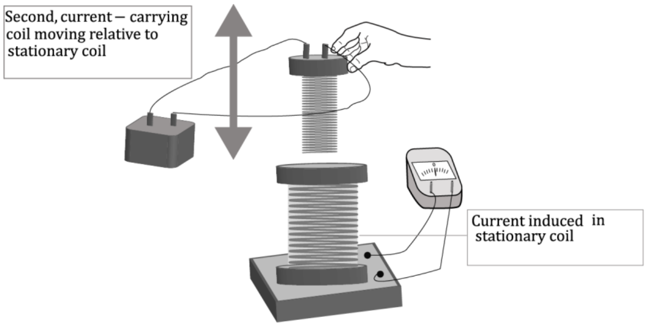
4. With the coil in a horizontal plane, we squeeze it so as to decrease the cross sectional area of the coil. The meter detects current only during the deformation, not before or after. When we increase the area to return the coil to its original shape, there is current in the opposite direction, but only while the area of the coil is changing.



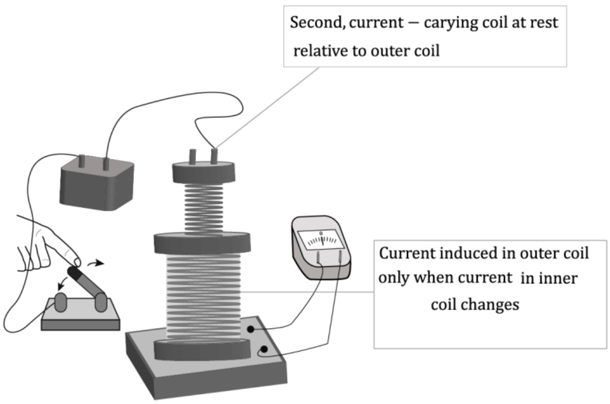
(a)



(b)



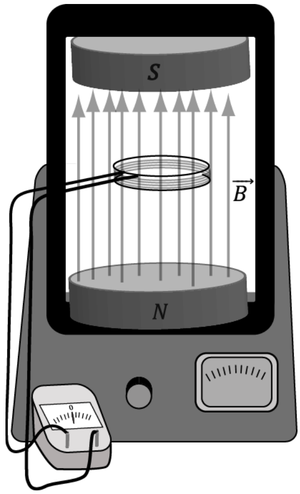
(c)



(d)

(a) A stationary magnet has no effect on a stationary coil of wire. A galvanometer connected to the coil shows zero current**.**
(b) When the magnet and coil move relative to each other, a current is induced in the coil. The current is in one direction if the magnet moves down and the opposite direction if the magnet moves up.
(c) We get the same effect as in (b) if we replace the magnet by a second coil carrying a constant current.
(d) When the switch is opened or closed, the change in the inside coil's current induces a current in the outer coil

5. If we rotate the coil a few degrees about a horizontal axis, the meter detects current during the rotation, in the same direction as when we decreased the area. When we rotate the coil back, there is a current in the opposite direction during this rotation
6. If we jerk the coil out of the magnetic field, there is a current during the motion, in the same direction as when we decreased the area.



A coil in a magnetic field. When the $\vec{B}$ field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil. A current is induced when any of these factors change.

7. If we decrease the number of turns in the coil by unwinding one or more turns, there is a current during the unwinding, in the same direction as when we decreased the area. If we wind more turns onto the coil, there is a current in the opposite direction during the winding

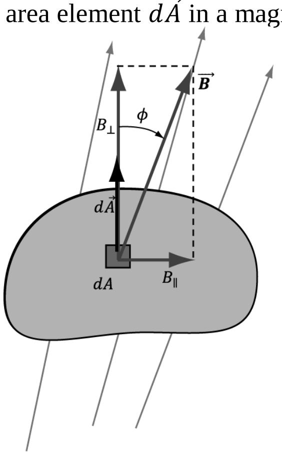
8. When the magnet is turned off, there is a momentary current in the direction opposite to the current when it was turned on

9. The faster we carry out any of these changes, the greater the current

10. If all these experiments are repeated with a coil that has the same shape but different material and different resistance, the current in each case is inversely proportional to the total circuit resistance. This shows that the induced emf that are causing the current do not depend on the material of the coil but only on its shape and the magnetic field

Faraday's Law

The common element in all induction effects is changing magnetic flux through a circuit. Before stating the simple physical law that summarizes all of the kinds of experiments described in section, let's first review the concept of magnetic flux ϕ_B (which we introduced in section). For an infinitesimal area element $d\vec{A}$ in a magnetic field $\vec{B}$, the magnetic flux $d\phi_B$ through the area is



The magnetic flux through an area element dA is defined to be $d\phi_B = B_\perp dA$

$$d\phi_B = \vec{B}.d\vec{A} = B_{\perp}dA = B\ dA\cos\phi$$

Where $B_{\perp}$ is the component of $\vec{B}$ perpendicular to the surface of the area element and ϕ is the angle between $\vec{B}$ and $d\vec{A}$. (be careful to distinguish between two quantities named "phi" ϕ and ϕ_B) The total magnetic flux ϕ_B through a finite area is the integral of this expression over the area

$$\phi_B = \oint \vec{B}\,.d\vec{A} = \int B\ \ dA\cos\phi$$

$$\phi_B = \vec{B}\vec{A} = BA\cos\phi$$

Faraday's law of induction states

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

In symbols, Faraday's law is

$$\epsilon = -\frac{d\phi_B}{dt}\ \text{(faraday's law of induction)}$$

To understand the negative sign, we have to introduce a sign convention for the induced emf ϵ. But first let's look at a simple of this law in action

Direction of Induced EMF

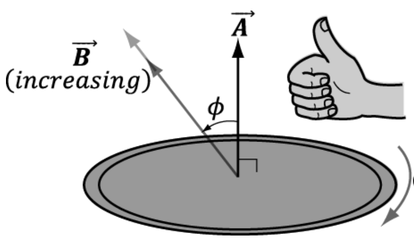
We can find the direction of an induced emf or current by using Eq. together with some simple sign rules. Here's the procedure

1. Define a positive direction for the vector area $\vec{A}$

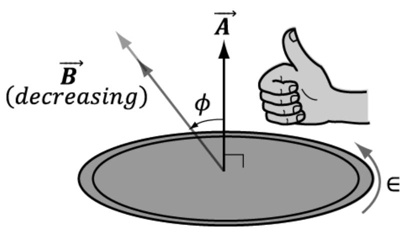
2. From the directions $\vec{A}$ of and the magnetic field $\vec{B}$, determine the sign of the magnetic flux ϕ_B and its rate of change $d\phi_B/dt$ Fig shows several examples

3. Determine the sign of the induced emf or current. If the flux is increasing, so $d\phi_B/dt$ is positive, then the induced emf or current is negative, if the flux is decreasing, $d\phi_B/dt$ is negative and the induced emf or current is positive.

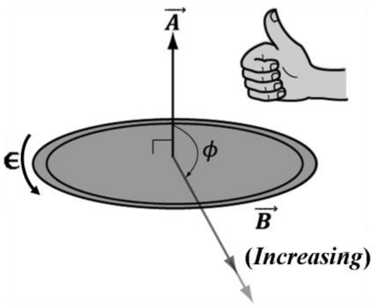
4. Finally, determine the direction of the induced emf or current using your right hand. Curl the fingers of your right hand around the vector $\vec{A}$, with your right thumb in the direction of $\vec{A}$. If the induced emf or current in the circuit is positive, it is in the same direction as your curled fingers; if the induced emf or current is negative, it is in the opposite direction.



(a)
Positive flux ($\phi_B > 0$)
Flux becoming more positive $\left(\frac{d\phi_B}{dt} < 0\right)$
Induced emf is negative (ε > 0)



(b)
Positive flux ($\phi_B > 0$)
Flux becoming less positive $\left(\frac{d\phi_B}{dt} < 0\right)$
induced emf is positive (ε > 0)

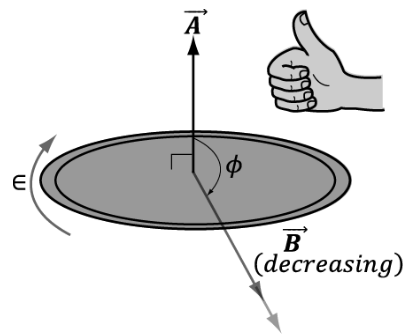


(c)

Negative flux ($\phi_B < 0$)

Flux becoming more negative $\left(\frac{d\phi_B}{dt} < 0\right)$

Induced emf is positive ($\varepsilon > 0$)



(d)

negative flux ($\phi_B < 0$)

Flux becoming less negative $\left(\frac{d\phi_B}{dt} > 0\right)$

induced emf is negative ($\varepsilon > 0$)

The magnetic flux is becoming (a) more positive, (b) less positive, (c) more negative, and (d) less negative. Therefore ϕ_B is increasing in (a) and (d) and decreasing in (b) and (c). In (a) and (d) the emfs are negative (they are opposite to the direction of the curled fingers of your right hand when your right thumb points along $\vec{A}$). In (b) and (c) the emfs are positive (in the same direction as the curled fingers).

Lenz's Law

Lenz's law is a convenient alternative method for determining the direction of an induced current or emf. Lenz's law is not an independent principle; it can be derived from Faraday's law. It always gives the same results as the sign rules we introduced in connection with Faraday's law, but it is often easier to use. Lenz's law also helps us gain intuitive understanding of various induction effects and of the role of energy conservation. H. F. E. Lenz (1804 – 1865) was a Russian scientist who duplicated independently many of the discoveries of Faraday and Henry. Lenz's law states

The direction of any magnetic induction effect is such as to oppose the cause of the effect. The "cause" may be changing flux through a stationary circuit due to a varying magnetic field, changing flux due to motion of the conductors that make up the circuit, or any combination. If the flux in a stationary circuit changes, the induced current sets up a magnetic field of its own within the area bounded by the circuit, this field is opposite to the original field if the original field is increasing but is in the same direction as the original field if the latter is decreasing. That is, the induced current opposes the change in flux through the circuit (not the flux itself)

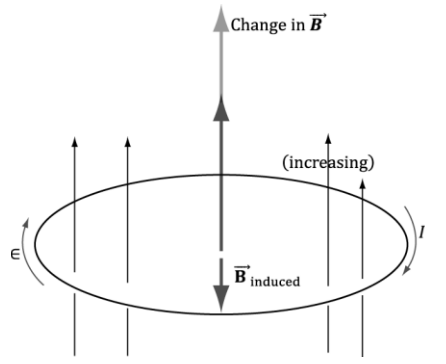
If the flux change is due to motion of the conductors, the direction of the induced current in the moving conductor is such that the direction of the magnetic field force on the conductor is opposite in direction to its motion. Thus the motion of the conductor, which caused the induced current, is opposed. We saw this explicitly for the slide wire generator in Ex. In all these cases the induced current tries to preserve the status quo by opposing motion or a change of flux.

Lenz's law is also directly related to energy conservation. If the induced current in Ex. Were in the direction opposite to that given by Lenz's law, the magnetic force on the rod would accelerate it to ever increasing speed with no external energy source, even though electric energy is being dissipated in the circuit. This would be a clear violation of energy conservation and doesn't happen in nature.

Finding the Direction of induced current

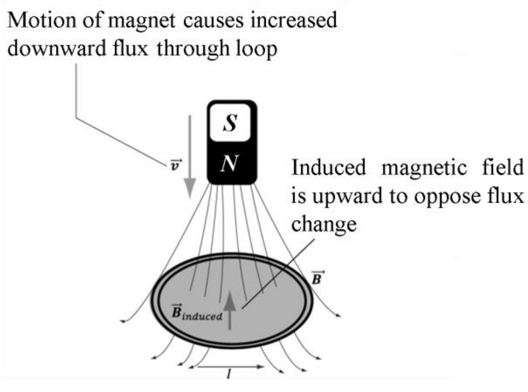
In Fig there is a uniform magnetic field $\vec{B}$ through the coil. The magnitude of the field is increasing, and the resulting induced emf causes an induced current. Use Lenz's law to determine the direction of the induced current.

This situation is the same as in Ex. By Lenz's law the induced current must produce a magnetic field $\vec{B}_{\text{induced}}$ inside the coil that is downward, opposing the change in flux. Using the right – hand rule we described in Section for the direction of the magnetic field produced by a circular loop, $\vec{B}_{\text{induced}}$ will be the desired direction if the induced current flows as shown in fig

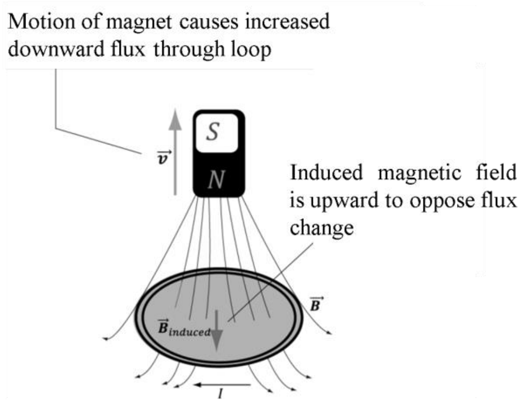


The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{\text{induced}}$ that it causes is downward, opposing the charge in the upward field $\vec{B}$.

Fig shows several application of Lenz's law to the similar situation of a magnet moving near a conducting loop. In each of the four cases shown, the induced current produces a magnetic field of its own, in a direction that opposes the change in flux through the loop due to the magnet's motion



(a) To produce this induced field, induced current must be counterclockwise as seen from above loop



(b) To produce this induced field, induced current must be clockwise as seen from above loop

Motion Electromotive Force

We've seen several situations in which a conductor moves in a magnetic field, as in the generators discussed in Ex. We can gain additional insight into the origin of the induced emf in these situations by considering the magnetic forces on mobile charges in the conductor. Fig shows the same moving rod that we discussed in Ex, separated for the moment from the U – shaped conductor. The magnetic field $\vec{B}$ is uniform and directed into the page, and we move the rod to the right at a constant velocity $\vec{v}$. A charged particle q in the rod then experiences a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ with magnitude $F = |q|vB$. We'll assume in the following discussion that q is positive; in that case the direction of this force is upward along the rod, from b toward a.

This magnetic force causes the free charges in the rod to move, creating an excess of positive charge at the upper end a and negative charge at the lower end b. this in turn creates an electric field $\vec{E}$ within the rod, in the direction from a toward b (opposite to the magnetic force). Charge continues to accumulate at the ends of the rod until $\vec{E}$ becomes large enough for the downward electric force (with magnitude qE) to cancel exactly the upward magnetic force (with magnitude qbB). Then qE = qvB and the charges are in equilibrium

The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric field magnitude E multiplied by the length L of the rod. From the above discussion, $E = vB$, so

$$V_{ab} = EL = vBL$$

With point a at higher potential than point b.

$$\epsilon = vBL$$

(motional emf; length and velocity perpendicular to $\vec{B}$ uniform)

$$d\epsilon = (\vec{v} \times \vec{B}).\vec{dl}$$

$$\epsilon = \oint(\vec{\boldsymbol{v}} \times \vec{B}).\vec{dl}$$

Illustration

In Fig. there is a + x – directed magnetic field of 0.2 T. Find the magnetic flux through each face of the box

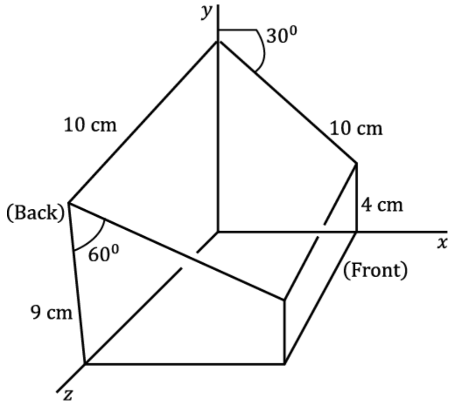
Solution

Let $\hat{n}$ represent the outward unit normal vector to a given face of the box and A the area of that face.

Then the outward flux through the face is $\phi = B.\hat{n}A = BA\cos\theta$. Clearly $\phi = 0$ through the two side faces ($\hat{n}$ in $\pm z$ direction) and the bottom face ($\hat{n}$ in $-y$ direction) through the front and back faces $\hat{n}$ is along +x and –x, respectively, so

$\phi_{\text{front}} = (0.2\text{ T})(40 \times 10^{-4}\text{ m}^2)$

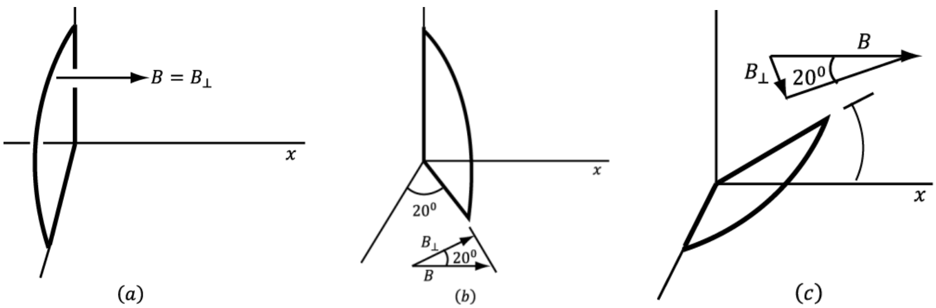
⇒ 0.8 m Wb



$\phi_{\text{back}} = (0.2\ \text{T})(90 \times 10^{-4}\ \text{m}^2)(-1)$
$\Rightarrow = -1.8\ \text{m Wb}$
(The minus sign indicates flux is inward through surface). For the top surface $\theta =$ 60° *and* $\phi_{\text{top}} = (0.2\ \text{T})(100 \times 10^{-4}\ \text{m}^{-2}) = 1.0\ \text{m Wb}$

Illustration

A loop of wire is placed in a magnetic $B = 0.0200iT$ field. Find the flux through the loop if its area vector is $A = 30i + 16j + 23k\ \text{cm}^2$. What is the angle between B and A



Solution

$\phi = B.A = (0.0200i\ T)(30i + 16j + 23k\ 10^{-4}\ \text{m}^2)$
$\Rightarrow 60\ \mu Wb$

In terms of magnitudes

$\phi = B.A\cos\theta$
$B = 0.0200\ \text{T}$

And

$A = (30^2 + 16^2 + 23^2)^{1/2} = 41.0\ \text{cm}^2$

Then

$6.0 \times 10^{-5}\ Wb$
$\Rightarrow (0.02\ T)(41 \times 10^{-4}\ \text{m}^2)\cos\theta$

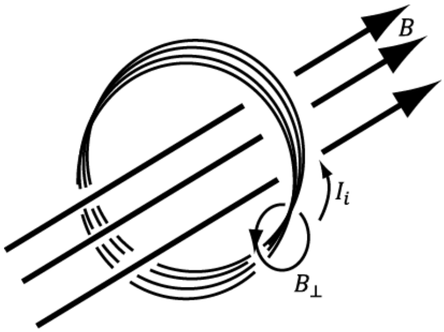
And

$$\cos\theta = 0.73 \text{ or } \theta = 43^o.$$

Illustration

The perpendicular component of the external magnetic field through a 10 – turn coil of radius 50 mm increases from 0 to 18 T in 3s, as shown in Fig. If the resistance of the coil is 2Ω, what is the magnitude of the induced current what is the direction of the current

Solution



The initial flux ϕ_1 is zero, and the final flux ϕ_2 is

$$\phi = NB_nA$$
$$\Rightarrow (10)(18\,T)(25\pi \times 10^{-4}\ \text{m}^{-2})$$
$$\Rightarrow 1.41\ Wb$$

And so

$$\Delta\phi = \phi_2 - \phi_1 = 1.41\,Wb$$

The induced emf if

$$\delta = -\frac{\Delta\phi}{\Delta t} = -\frac{1.41\,Wb}{3s}$$
$$\Rightarrow\ -0.47\,V$$

The minus sign indicates that the emf will cause a current that creates a field opposing the change in B. the magnitude of the induced current is

$$I_1\frac{\delta}{R} = \frac{0.47}{2\Omega}$$
$$\Rightarrow 0.235\,A.$$

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