Chapter 2

Magnetic field of Current Element

Day – 1

Introduction

Principle of superposition of magnetic fields: The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges. We begin by calculating the magnetic field caused by a short segment $d\vec{t}$ of a current – carrying conductor, as shows in Fig. The volume of the segment is a dl, where A is the cross – sectional area of the conductor. If there are moving charged particles per unit volume, each of charge q, the total moving charge dQ in the segment is

$$dQ = nqA\,dl$$

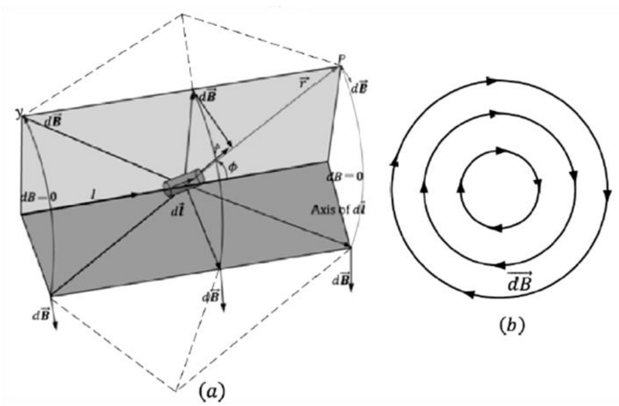
The moving charges in this segment are equivalent to a single charge dQ, traveling with a velocity equal to the drift velocity $\vec{v_d}$. (Magnetic fields due to the random motions of the charges will, on average, cancel out at every point) From the magnitude of the resulting field $\overrightarrow{dB}$ at any field point P is

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d \,\mathrm{A}\, dl \sin\phi}{r^2}$$

But from Eq. $n|q|v_d$ A equals the current I in the element. So

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin\phi}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \text{ (Magnetic field of a current element)}$$



For these field points, $\vec{r}$ and $d\vec{l}$ both lie in the tan – colored plane, and $d\vec{B}$ is perpendicular to this plane for these field points, $\vec{r}$ and $d\vec{l}$ both lie in the orange – colored plane, and $d\vec{B}$ is perpendicular to this plane

(a) Magnetic – field vectors due to a current element $d\vec{l}$. (b) Magnetic field lines in a plane containing the current element $d\vec{l}$**.** The × indicates that the current is directed into the plane of the page. Compare this figure to Fig. for the field of a moving point charge.

Law of Biot and Savart (pronounced "Bee – oh" and "Such – var"). We can use this law to find the total magnetic field $\vec{B}$ at any point in space due to the current in a complete circuit. To do this, we integrate over all segments $d\vec{l}$ that carry current; symbolically,

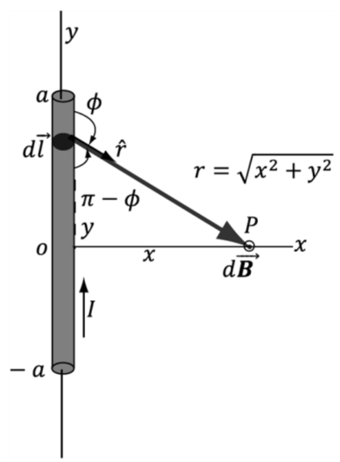
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

The field vectors $d\vec{B}$ and the magnetic field lines of a current element are exactly like those set up by a positive charge dQ moving in the direction of the drift velocity $\vec{v_d}$. The field lines are circles in planes perpendicular to and $d\vec{l}$ centered on the line of $d\vec{l}$. Their directions are given by the same right – hand rule that we introduced for point charges.

What we measure experimentally is the total $\vec{B}$ for a complete circuit. But we can still verify these equations indirectly by calculating $\vec{B}$ for various current configurations using and comparing the results with experimental measurements.

If matter is present in the space around a current – carrying conductor, the field at a field point P in its vicinity will have an additional contribution resulting from the magnetization of the material. We‟ll have return to this point in Section .However, unless the material is iron or some other ferromagnetic material, the additional field is small and is usually negligible. Additional complications arise if time – varying electric or magnetic fields are present or if the material is a super – conductor; we‟ll return to these topics later.

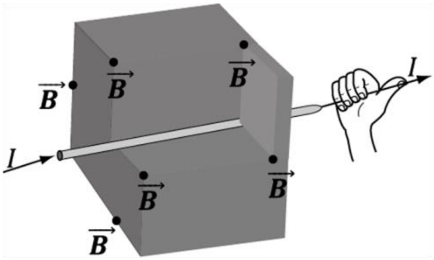
Magnetic Field of a Straight Current- Carrying Conductor



Magnetic field produced by a straight current-carrying conductor of length 2a. At point P, $\vec{B}$ is directed into the plane of the page

We first use the law of Biot and Savart, to find the field $d\vec{B}$ caused by the element of conductor of length dl = dy. From the figure, $r = \sqrt{x^2 + y^2}$ and $\sin\phi = \sin(\pi - \phi) = \frac{x}{\sqrt{S^2+y^2}}$. The right- hand rule for the vector product $d\vec{l} \times \hat{l}$ shows that the direction of $d\vec{B}$ is into the plane of the figure, perpendicular to the plane; furthermore, the directions of the $d\vec{B}s$, a significant simplification

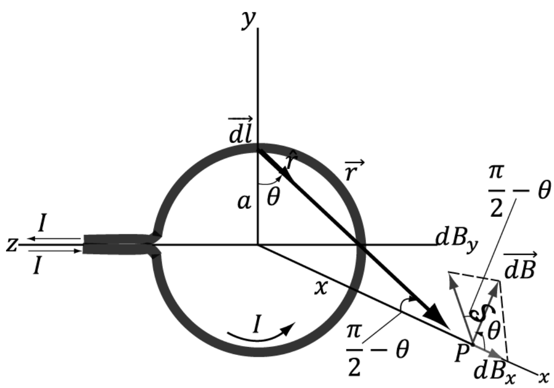
$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x\,dy}{(x^2+y^2)^{3/2}}$$
$$B = \frac{\mu_0 I}{4\pi} - \frac{2a}{x\sqrt{x^2+a^2}}$$
$$B = \frac{\mu_0 I}{2\pi x}$$
$$B = \frac{\mu_0 I}{2\pi r} \text{ (a long straight, current − carrying conductor)}$$



Magnetic field around a long, straight, current – carrying conductor. The field lines are circles, with directions determined by the right – hand rule.

Magnetic Field of a Circular Current Loop

Magnetic field of a circular loop. The current in the segment $d\vec{l}$ causes the field $d\vec{B}$, which lies in the xy-plane. The currents in other $d\vec{l}'s$ cause $\overrightarrow{dB's}$ with different components perpendicular to the x – axis; these components add to zero. The x - components of the $\overrightarrow{dB's}$ combine to give the total $\vec{B}$ field at point P

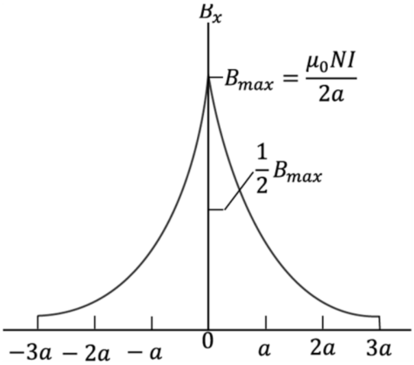


$$dB = \frac{\mu_0}{4\pi} \frac{dl}{(x^2+a^2)}$$
$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+a^2)} \frac{a}{(x^2+a^2)^{1/2}}$$
$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2+a^2)} \frac{x}{(x^2+a^2)^{1/2}}$$

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a\, dl}{(x^2+b^2)^{3/2}} = \frac{\mu_0 l_a}{4\pi(x^2+a^2)^{3/2}} \int dl$$

$$B_x = \frac{\mu_0 I a^2}{2(x^2+b^2)^{3/2}} \text{ (on the axis of a circular loop)}$$

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+b^2)^{3/2}} \text{ (on the axis of N circular loop)}$$



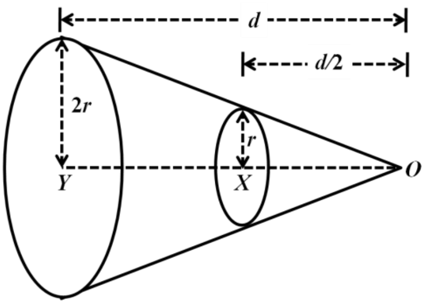
Graph of the magnetic field along the axis of a circular coil with N turns. When x is much larger than a, the field magnitude decreases approximately as 1/x3

$$B_x = \frac{\mu_0 N I}{2a} \text{(at the center of N circular loops)}$$

$$B_x = \frac{\mu_0 N}{2\pi(x^2+a^2)^{3/2}} \text{ (on the axis any number of circular loops)}$$

Illustration

Two circular coils X and Y having equal number of turns and carry equal currents in the same sense and subtend same solid angle at point O. If the smaller coil X is midway between O and Y, then if we represent the magnetic induction due to bigger coil Y at O as B, and due to smaller coil X at O s B_x then



(a) $B_y/B_x = 1$ (b) $B_y/B_x = 2$
(c) $B_y/B_x = 2$ (d) $B_y/B_x = ¼$

Solution

$$B_y = \frac{\mu_0}{4\pi} \frac{2\pi i (2r)^2}{((2r)^2+d^2)^{\frac{3}{2}}}$$

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi i(r)^2}{\left(r^2+\frac{d^2}{4}\right)^{3/2}}$$

Now use binomial $(1 + x)^n = 1 + nx$ if $x << i$ then divide.

Illustration

Two circular coils of wires made of similar wires but of radius 20 cm and 40 cm are connected in parallel. The ratio of the magnetic fields at their centre is

(a) 4 : 1 (b) 1 : 4
(c) 2 : 1 (d) 1 : 2

Solution

$$\frac{i_1}{i_2} = \frac{R_1}{R_2}$$

$$\Rightarrow \quad \frac{l\left(\frac{l_2}{A}\right)}{l\left(\frac{l_1}{A}\right)} = \frac{l_2}{l_1}$$

$$\frac{i_1}{i_2} = \frac{2\pi r_2}{2\pi r_1}$$

$$\frac{B_1}{B_2} = \frac{\left(\mu_0\frac{i_1}{2r_1}\right)}{\left(\mu_0\frac{i_2}{2r_2}\right)}$$

$$\Rightarrow \quad = \left(\frac{r_2}{r_1}\right)^2$$

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