Chapter 11 Gravitation

Day - 1

NEWTON'S LAW OF GRAVITATION

$F \propto m_1 m_2 \propto \frac{1}{r^2}$

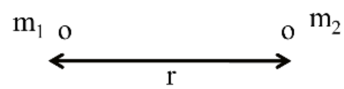
$\Rightarrow F = G\frac{m_1 m_2}{r^2}$

Here G is universal gravitational Constant

G = 6.67 x 10^{-11} Nmt^2/kg^2

Dimensional formula = $[M^{-1} L^3 T^{-2}]$

Note: This formula is applicable for point mass only.



PROPERTIES OF GRAVITATIONAL FORCE

(1) It is always only attraction type force never repel.

(2) It is independent of medium between particles

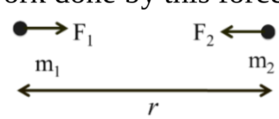
(3) It is mutual force

$|F_1| = |F_2| = \frac{Gm_1m_2}{r^2}$

$\frac{F_1}{F_2} = \frac{1}{1}, \vec{F}_1 = -\vec{F}_2$

(4) It is applicable for very far distant objects like interplanetary distance as well as for very short distance like inter-atomic distance

(5) It is conservative force means work done by this force is independent of path.



Example: Two particles of equal masses move in a Circle of radius r under the action of mutual gravitational attraction force. Then the speed of each particle if the mass of each particle is m .

(a) $\sqrt{\frac{Gm}{r}}$ (b) $\sqrt{\frac{Gm}{2r}}$

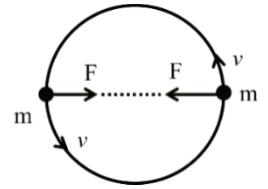
(c) $\sqrt{\frac{Gm}{4r}}$ (d) $\sqrt{\frac{2Gm}{r}}$

Solutions:

Here gravitational force F is centripetal force provider

$F = \frac{Gmm}{(2r)^2} = \frac{mv^2}{r}$

$\Rightarrow V = \sqrt{\frac{Gm}{4r}}$

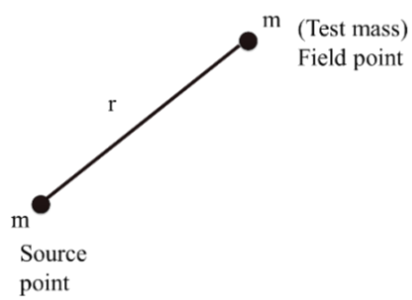


Answer (c)

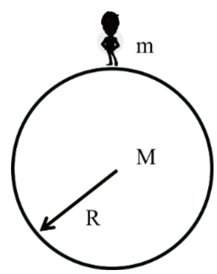
GRAVITATIONAL FIELD

Space of influence surrounding a mass m in which its gravitational effects are effective is called gravitational field of given mass.

Intensity of gravitational field due to point mass



GRAVITATIONAL FIELD OF EARTH HERE WE ARE ASSUMING



(1) Shape of earth is perfect sphere

(2) Center of mass of earth is at its center

(3) Uniform mass distribution

$F = \frac{GMm}{R^2}$

Now $F = \frac{GMm}{R^2}$

$E_g = \frac{F}{m} = \frac{GMm}{mR^2} = \frac{GM}{R^2}$

Intensity of earth gravitational field at its surface

Now here $E_g = g$ we will use another term specially for earth is acceleration due to gravity Now At earth's Surface $g = \frac{GM}{R^2}$ Direction of E_g or g is always towards the Center of earth Also weight of any object at the surface of earth W = F = m E_g = mg Force acting towards the center of earth Properties of 'g'

(1) It is independent of mass of any object

(2) It is not a universal constant its value depends on place, position and planet.

NOTE: For example value of g on the surface of moon is approximately g/6 i.e. 1/6 of the value on the earth

Variation of 'g'

(1) Due to change of planet

As we know at the surface of earth

$g = \frac{GM}{R^2}$, Here density $\rho = \frac{M}{\text{Volume}}$

$\Rightarrow M = \rho \frac{4}{3}\pi R^3$

$\Rightarrow g = \frac{G}{R^2} \times \rho \times \frac{4}{3}\pi R^3$

$\Rightarrow g = \frac{4}{3} G\rho\pi R$

(2) Due to shape of earth

The earth is not perfectly spherical in shape but is on oblate sphere. It is bulged at the equator and flattened at the pole.

Equatorial radius is 21 km more than polar radius

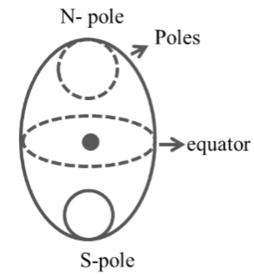
As we know $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$g \rightarrow$ Minimum at equator

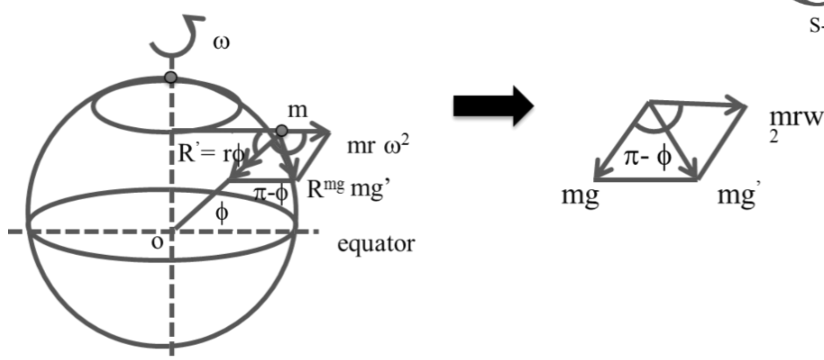
$g \rightarrow$ Maximum at poles

At the surface

$g_{poles} > g_{equator}$



(3) DUE TO ROTATION OF EARTH



USING LAW OF PARALLELOGRAM

$R^2 = P^2 + Q^2 + 2\,PQ\cos\alpha$

$(mg')^2 = (mg)^2 + (mr\omega^2)^2 + 2\,mg\,mr\omega^2\cos(\pi - \phi)$

$g^1 = \sqrt{g^2 + r^2\omega^4 - 2gr\omega^2\cos\phi}$

$g^1 = \sqrt{g^2 + R^2\omega^4\cos^2\phi - 2gR\omega^2\cos^2\phi}$

$g^1 = g\sqrt{1 + \frac{R^2\omega^4}{g^2}\cos^2\phi - \frac{2R\omega^2}{g}\cos^2\phi}$

Now Here $\frac{R^2\omega^2}{g^2}$ will be neglected

$g' = g\left(1 - \frac{2R\omega^2}{g}\cos^2\emptyset\right)^{1/2}$

$\Rightarrow g' = g - R\omega^2\cos^2\emptyset$

At the equator $\phi = 0°$

At the poles $g' = g - R\omega^2$

$\phi = 90°$, $g^1 = g - R\omega^2\cos^2 90°$ $g' = g$

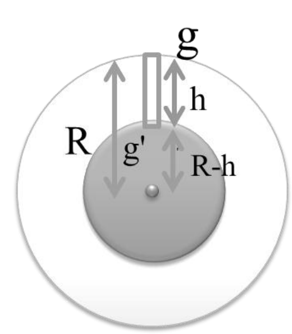
4. At the depth h inside the earth

At the surface of earth

$$g = \frac{GM}{R^2}$$

At depth h

$$g' = \frac{GM'}{(R-h)^2}$$



Here

$\frac{4}{3}\pi R^3 - - - M$

$\frac{4}{3}\pi (R-h)^3 - - - \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi (R-h)^3$

$= \frac{M(R-h)^3}{R^3} = m'$

$g' = \frac{G}{(R-h)^2}\frac{M(R-h)^3}{R^3}$

$g' = \frac{GM}{R^2}\left(\frac{R-h}{R}\right) \Rightarrow g' = g\left(1 - \frac{h}{R}\right)$

At the Center of the earth

$h = R,\ g' = g\ \left(1 - \frac{R}{R}\right)$

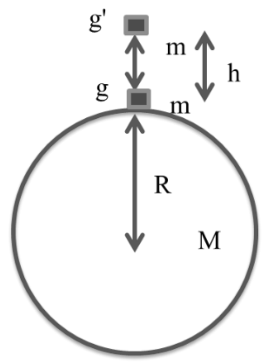
g' = 0 weight less

5. At *h* height above the earth's surface

At the surface of earth

$g = \frac{GM}{R^2}$

At the h height above



$g' = \frac{GM}{(R+h)^2}$

$g' = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$

$g' = g\left(1 + \frac{h}{R}\right)^{-2}$

$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$g' = g\ \left(1 - \frac{2h}{R}\right)$ Here

If h << R

Binomial expansion

$(1+ x)^n = 1+ nx$

If x <<<<<<1