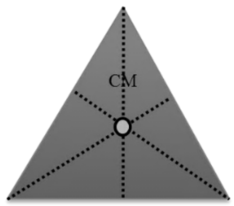
Chapter 9 Center of mass & Momentum

Dav - 1

CENTRE OF MASS AND MOMENTUM

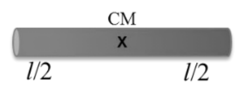
Centre of mass

It is a point of a body or system at which the whole mass of body or system is supposed to be Concentrated for dealing its translation motion.



Equilateral lamina

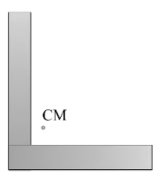
Square lamina



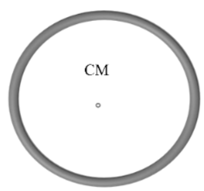
Rod with uniform mass distribution

Properties of center of mass

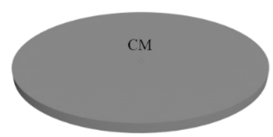
(1) Three may or may not be any mass present physically at center of mass. So its position depends on shape of the body.



L-Shaped Strips

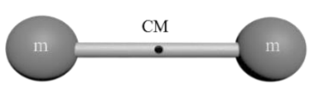


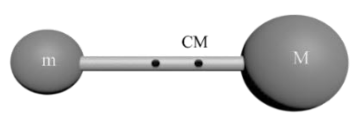
Ring



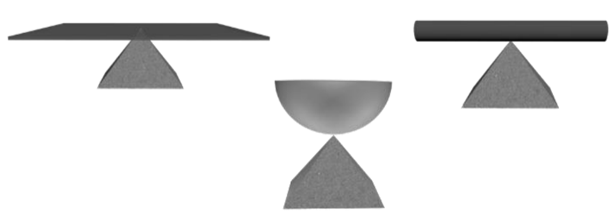
Uniform disc

(2) For a given shape it depends on the distribution of mass within the body and is closer to massive part.





(3) If we apply the external force on cm then we will get maximum results



TO LOCATE CENTRE OF MASS

x- Coordinate of CM

$$x_{cm} = \frac{m_1x_1m_2x_2+m_3x_3+-----}{m_1+m_2+-----}$$

y-Co-ordinate of CM

$y_{cm} = \frac{m_1y_1+m_2y_2+-----}{m_1+m_2+-----}$

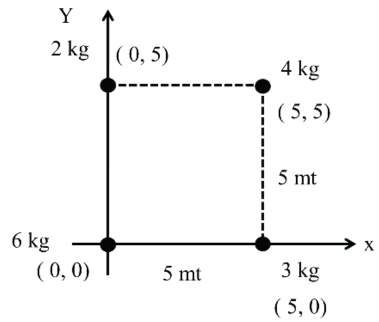
z-Co- ordinate of CM

$Z_{cm} = \frac{m_1z_1+m_2z_2+m_3z_3+-----}{m_1+m_2+m_3+--}$

Similarly in terms of position Vector

$r_{cm} = \frac{m_1r_1+m_2r_2+-----}{m_1+m_2+--}$

Example



Step1. Choose your origin (try to fix your origin at extreme left and lower end)

Step 2. Use above results e.g. x_{cm}, y_{cm} etc.

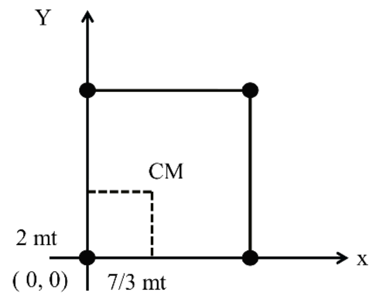
$x_{cm} = \frac{6\times0+3\times5+4\times5+2\times0}{6+3+4+2}$

$= \frac{35}{15} = \frac{7}{3} met$

$y_{cm} = \frac{6\times0+3\times0+4\times5+2\times5}{6+3+4+2}$

$= \frac{30}{15} = 2met$

$(x_{cm}, y_{cm}) = \left(\frac{7}{3}, 2\right)$



POSITION OF CENTRE OF MASS Arc of thin ring

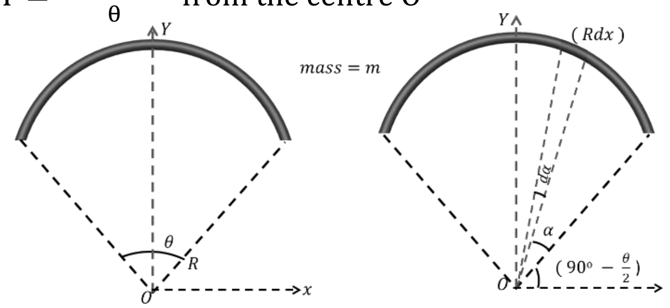
$dm = \frac{m}{\theta}\, d\alpha$ and $Y = R \sin\left(\alpha + 90° - \frac{\theta}{2}\right)$

$Y_{cm} = \frac{\int y dm}{\int dm} = \frac{\int_0^\theta R \sin\left(90°+\alpha - \frac{\theta}{2}\right)\frac{m}{\theta} d\alpha}{\int_0^m dm}$

$Y_{cm} = \frac{R}{\theta}\int_0^\theta \cos\left(\alpha - \frac{\theta}{2}\right) d\alpha$

$= \frac{R}{\theta}\left[\sin\left(\alpha - \frac{\theta}{2}\right)\right]_0^\theta = \frac{R}{\theta} \times 2 \sin\theta/2$

COM of the are is at $Y = \frac{2R \sin\theta/2}{\theta}$ from the centre O



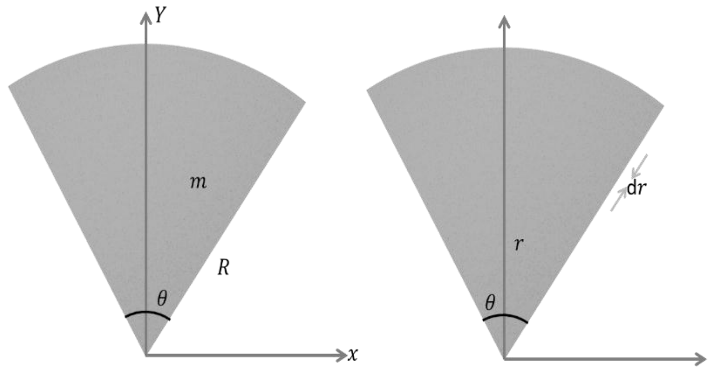
PART OF THE DISC

$Y = \frac{2r \sin\theta/2}{\theta}$ $dm = \frac{m}{\frac{1}{2}R^2\theta}\theta r\, dr = \frac{2mr}{R^2}dr$

Now $Y_{cm} = \frac{\int Y dm}{\int dm} = \frac{\int_0^R 2r\frac{\sin\theta/2}{\theta}\times\frac{2mr}{R^2}dr}{m}$

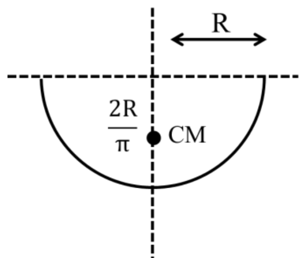
$= \frac{4\sin\theta/2}{R^2\theta}.\int_0^R r^2 dr$

$Y_{cm} = \frac{4R\sin\theta/2}{3\theta}$

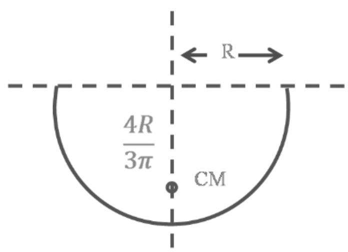


LOCATION OF C.M OF SOME HOMOGENEOUS RIGID BODIES

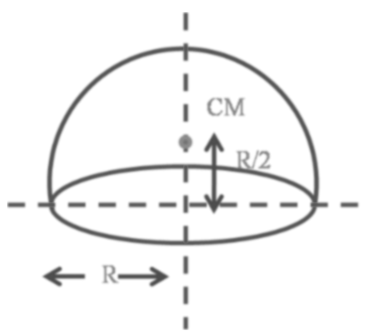
(1) Semi Circular ring



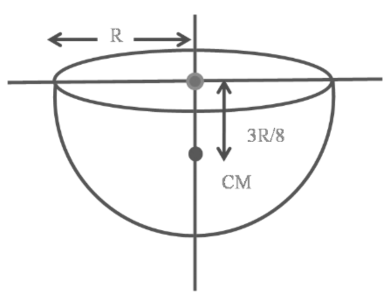
(2) Semi Circular disc



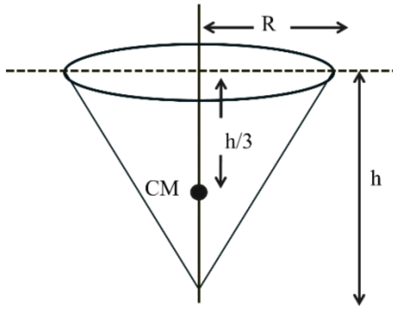
(3) Hollow hemisphere



(4) Solid hemisphere



(5) Cone (Hollow)



(6) Solid cone