Chapter 7 Circular motion

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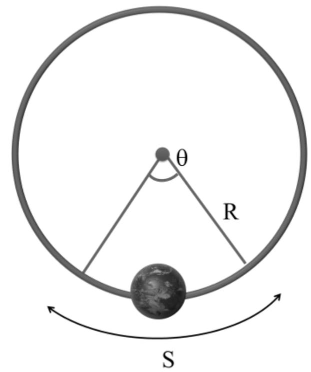
If Any object moves such that it covers an angle θ at the fixed point (center of the circle) and its gap from the fixed point remains constant (radius of the circle)



now angle $= \frac{\text{arc}}{\text{R}} = \frac{\text{S}}{\text{R}}$

$\theta = \frac{\text{S}}{\text{R}}$

Here R is constants

$\frac{d\theta}{dt} = \frac{1}{R}\frac{ds}{dt}$

$\omega = \frac{1}{R}V \Rightarrow v = R\omega$ In vector form $\vec{V} = \vec{\omega} \times \vec{R}$

Now finding linear speed of

(i) hour hand (30 cm)

(ii) Minute hand (90 cm)

(iii) Second hand (60 cm)

Solution:- (i) time period of hour hand T = 12 hr

= 12 x 3600 sec

v = r ω

$v = 30 \times \frac{2\pi}{T} = 30 \times \frac{2\pi}{12 \times 3600} = \frac{\pi}{720}$ cm/sec

(ii) Minute hand:- time period = T = 60 min = 60 x 60 sec

$v = r\omega\ 60 \times \frac{2\pi}{60 \times 60} = \frac{\pi}{30}$ cm/sec

(iii) Second hand:-

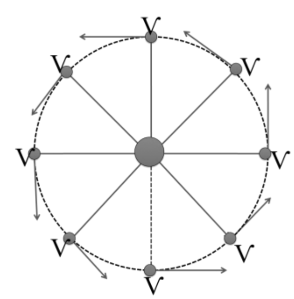
Time period= T = 60 sec

v = r ω

$= 90 \times \frac{2\pi}{60} = 3\pi\ cm/sec$

Types of circular motion: (on the basis of speed)

(1) Uniform circular motion: **(U C M)**



Only direction of velocity is changing, magnitude remains unchanged.

$|v| = \text{const}$

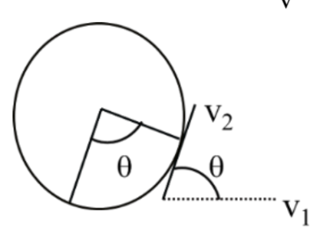
Now acceleration due to change in direction of velocity

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

Using law of parallelogram

$|\vec{v}^2| = v_1^2 + v_2^2 - 2v_1v_2\cos\theta$

But in UCM $|v_1| = |v_2| = v$



$|\Delta\vec{v}|^2 = v^2 + v^2 - 2v^2\cos\theta$

$= 2v^2 - 2v^2\cos\theta$

$= 2v^2(1 - \cos\theta)$

$|\Delta\vec{v}|^2 = 2v^2(1 - 1 + 2\sin^2\theta/2)$

$|\Delta\vec{v}|^2 = 4v^2(\sin^2\theta/2)$

$|\Delta\vec{v}|^2 = 2v\sin\frac{\theta}{2}$

Now

$|\Delta\vec{v}| = 2\ v\sin\frac{\theta}{2}$

$|\vec{a}| = \left|\frac{2\vec{v}}{\Delta t}\right| = \frac{2v}{t}\sin\frac{\theta}{2}$

$\sin\theta \approx \theta$ if $\theta <<<<$

$|\vec{a}| = \frac{2v}{t}\frac{\theta}{2}$

$a = v\frac{\theta}{t}$

$a = v\,\omega$,

Now $\omega = \frac{v}{R}$

$a = \frac{v^2}{R}$

or $a = R\omega^2$

Direction of this acceleration is towards center

Now $a_c = a_R = \frac{V^2}{R} = R\omega^2$

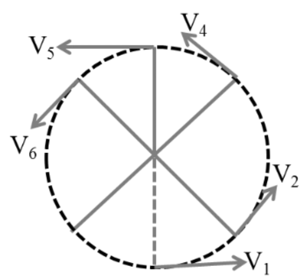
Centripetal acceleration or Radial acceleration

(2) Non uniform circular motion (Non U C M)

Here in this case direction as well as magnitude both are changing continuously therefore here two different named acceleration will be as,

(i) $a_c = a_R$ (centripetal or radial acceleration)

(ii) a_t (Tangential acceleration)



Now,

Tangential acceleration.

"Rate of change of magnitude of velocity"

$v = R\,\omega$

$\left\{\alpha = \frac{d\omega}{dt}\right\}$ Angular acceleration

$\frac{dv}{dt} = R\frac{d\omega}{dt}$

$a_t = \frac{dv}{dt} = R\alpha$

So $a_t \perp a_c$

$a_{net} = \sqrt{a_c^2 + a_t^2}$

Net acceleration for non-uniform circular motion.

Example:- A particle moving in a circular path of radius 2 meter and its velocity varies as $v = 10t^2$. Then net acceleration of the particle at t = 2 sec.

Solution: Given $v = 10t^2$

$a_t = \frac{dv}{dt} = 20t$

$= 20 \times 2$

$= 40\ m/s^2$

$a_c = \frac{v^2}{R} = \frac{10(2)^2}{2}$

$a_{net} = \sqrt{(40)^2 + (20)^2} = 20\sqrt{5} m/s^2$

Centripetal force:- the force require to move in a circular path for any object with respect to inertial frame is called centripetal force.

$F_c = F_R = \frac{mv^2}{R} = MR\omega^2$

Note:- Centrifugal force will also have same magnitude but direction opposite to centripetal force.

Direction of this force is towards the center.

HORIZONTAL CIRCULAR MOTION

(1) Only banking

Here N $\sin\theta$ is C.P. provider

So $N \sin\theta = \frac{mv^2}{R}$

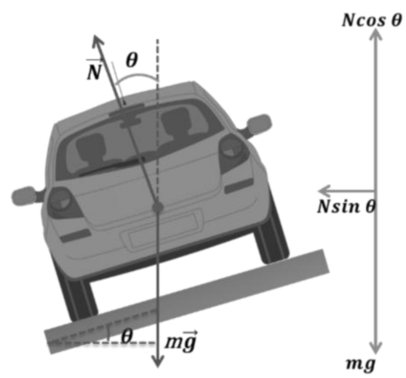
But N $\cos\theta$ = mg

$\tan\theta = \frac{v^2}{Rg}$

$v \leq Rg \tan\theta$

↓

Safe speed



(2) Only friction

Here now μ N is C.P. provider

$\mu N = \frac{mv^2}{R}$

N = mg

$\mu = \frac{v^2}{Rg}$

$v = \sqrt{\mu Rg}$

$v \leq \sqrt{\mu Rg}$

↓

Safe speed

Friction and Banking both:- To prevent inward sliding

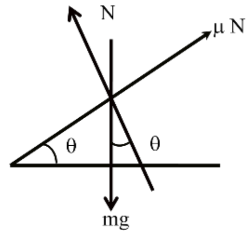
$N\cos\theta + \mu N\sin\theta = mg$

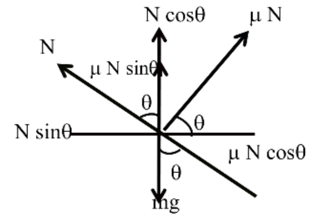
$N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R}$

Divide

$\frac{\cos\theta+\mu\sin\theta}{\sin\theta-\mu\cos\theta} = \frac{gR}{v^2}$

$v_1 = \frac{gR(\sin\theta-\mu\sin\theta)}{(\text{Cos}\theta+\mu\sin\theta)}$





TO PREVENT OUTWARD SLIDING

$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{R}$

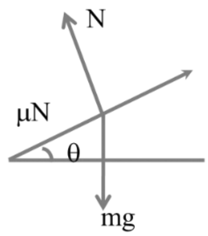
$N\cos\theta - \mu N\sin\theta = mg$

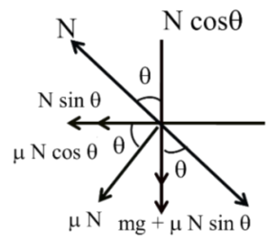
$\frac{sin\theta+\mu cos\theta}{cos\theta-\mu sin\theta} = \frac{v^2}{Rg}$

$v_2 = \sqrt{\frac{Rg(\sin\theta+\mu\cos\theta)}{(\cos\theta-\mu\sin\theta)}}$

$v_1 < V_{safe} < v_2$

$\sqrt{\frac{Rg(\tan\theta-\mu)}{(1+\mu\tan\theta)}} < v_{safe} < \sqrt{\frac{Rg(\tan\theta+\mu)}{(1-\mu\tan\theta)}}$





CONICAL PENDULUM

$T\sin\theta = mr\omega^2$

$T\cos\theta = mg$

$\tan\theta = \frac{r\omega^2}{g}$

$\omega = \sqrt{\frac{g\tan\theta}{r}}$

$\frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{r}}$

Time period of conical pendulum

$T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

Now $\sin\theta = \frac{r}{l} \Rightarrow r = l\sin\theta$

$T = 2\pi\sqrt{\frac{l\sin\theta\cos\theta}{g\sin\theta}} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$