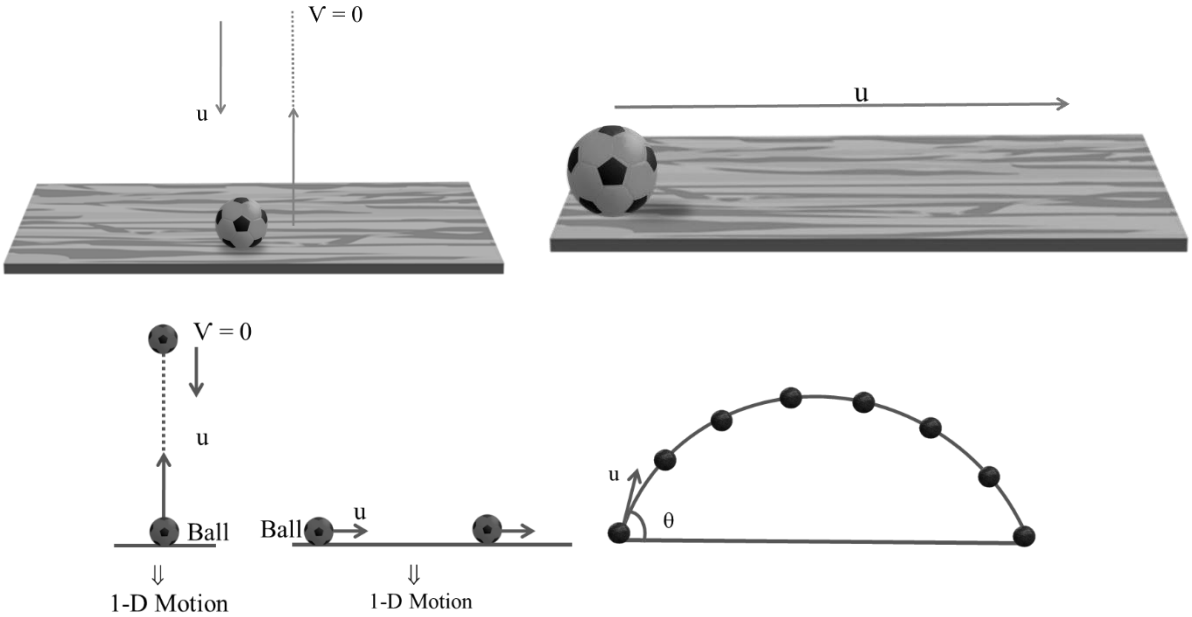
# Chapter 5 Projectile Motion

**Dav - 1**

## 1-D MOTION

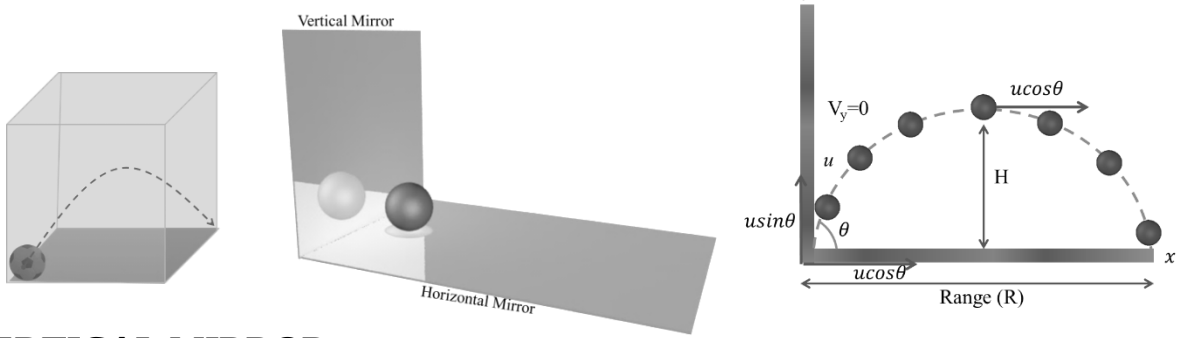



## 2-D MOTION OR MOTION IN A PLANE ⇒

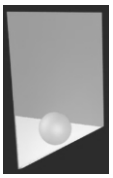
Motion of any object in any of two axis involve (xy or yz or zx)

"Projectile Motion may be 2-D or even 3-D also but in general it is use to be 2-D Motion"

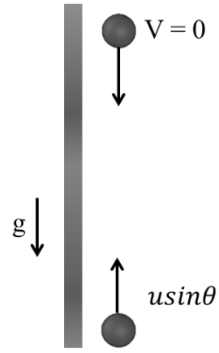
## 3-D PROJECTILE MOTION



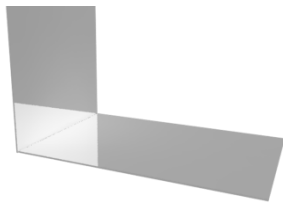

## VERTICAL MIRROR



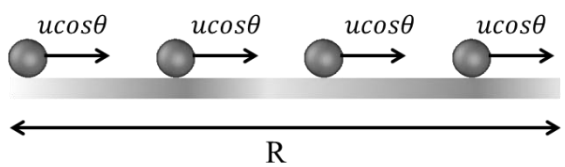
Vertical Mirror → Gravity acts in vertical direction so we can use equations of motion
Time taken by the image of ball going up = time taken going down = t (Let)
Use v = u – g t
O = u sinθ - g t
Total time
$t = \frac{u\sin\theta}{g}$, One sided time up or down
$T = 2t = \frac{2u\sin\theta}{g} \Rightarrow$ Time of flight
Now to get Maximum height
Using $v^2 = u^2 - 2g\,h$
$(O)^2 = (u\sin\theta)^2 - 2g\,H$
$H = \frac{u^2\sin^2\theta}{2g} \Rightarrow$ Maximum height




## HORIZONTAL MIRROR



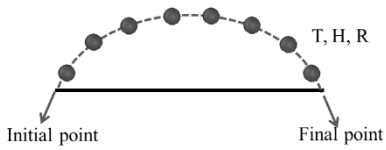
Velocity remains constant so we can not use equation of Motion
**Note:** Untill any external reason present to change ucosθ, like air flow ucosθ remains Constant
Now ucosθ = Const
$t = \frac{u\sin\theta}{g}$, one sided time up o
R = (u cosθ) T
$\frac{u\sin\theta}{g} \Rightarrow$ Time of flight
Now Sin2θ = 2 sinθ Cosθ
$T = 2t = \frac{2\,u\sin\theta}{g} \Rightarrow$ Time of flight
Now we have
$H = \frac{u^2\sin^2\theta}{2g} \Rightarrow$ Maximum hight
So $R = \frac{u^2\sin 2\theta}{g} \Rightarrow$ Range

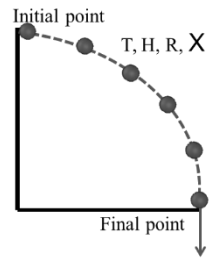



1. $T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$
2. $H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$
3. $R = \frac{u^2\sin 2\theta}{2g} = \frac{u_y^2}{2g}$

## CONDITION

"These three results only when can be use if initial point of projection and final point of projection are at same level.




**Example**: A body of mass m is projected upward with initial velocity $u\cos\theta = \frac{R}{T}$ then find time of flight, Maximum height attained and range attained by the body (g = 10m/s$^2$)

**Solution**

$R = (u\cos\theta)\frac{(2u\sin\theta)}{g}\,m/s$

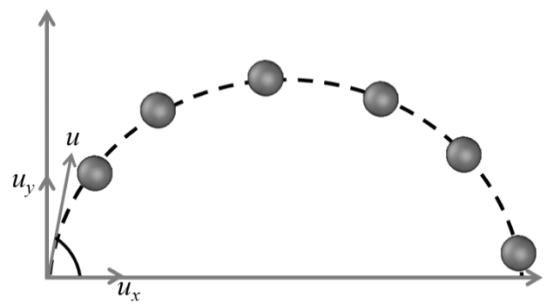
$u_x = 4\ m/s$

$u_y = 5m/s$

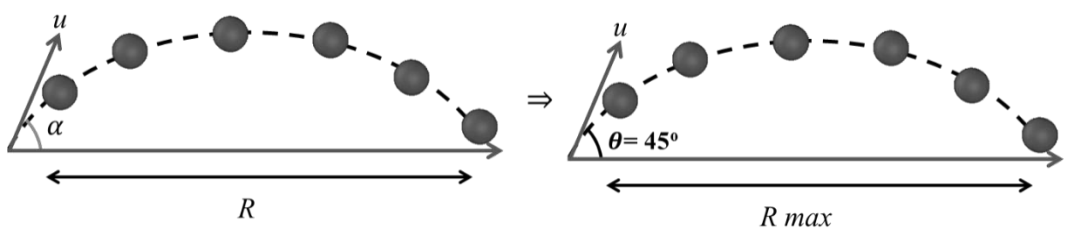
So $R = u^2\sin = \frac{2\times5}{10} = 1sec$

$H = \frac{u_y^2}{2g} = \frac{(5)^2}{2\times10} = \frac{25}{20} = 1.25$ meter

$2.\,H = \frac{u^2 Sin^2\theta}{2g} = \frac{u_y^2}{2g}\,u\sin\theta = 2$




**Maximum range:** To get maximum horizontal distance covered by any mass in projectile motion we must have a unique specified angle.




$R = \frac{u^2\sin2\theta}{g}$, Now for $R_{max} \rightarrow (Sin2\theta)_{max}$

$(Sin2\theta)_{max} = 1$

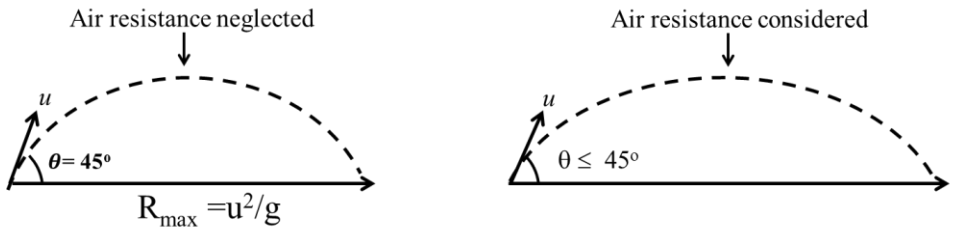
$2\theta = 90°$

$\theta = 45°$

To get maximum range angle of projection should be 45 °, $R_{max} = \frac{u^2}{g}$

In this case maximum height $= \frac{u^2\sin^2 45}{2g} = \frac{u^2}{4g}$

**Note**: Here we are neglecting the effect of air resistance. If we Consider air resistance then this angle θ should be little bit less than 45°




## SAME RANGE

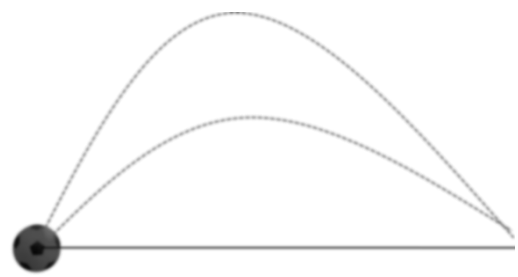
Mathematically there must be two different angle of projection for which we will get same range

Ball 1

Now $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1\text{ sec}$

Ball 2

$= \frac{2\times5}{10} = 1\text{ sec}$



Now if we assume

$\alpha + \beta = 90°$

then $\beta = 90° - \alpha$

$\vec{u} = (4\hat{i} + 5\hat{j})$

$R = \frac{2ux\ u_y}{g} = \frac{2\times4\times5}{10} = 4\text{ meter}$

$R = \frac{2u_x 2u_y}{g}$

If sum of angle of projection is = 90° and initial speed is same then both balls will have same range.

Note: Here in this Case only range will be same not time of flight and maximum height

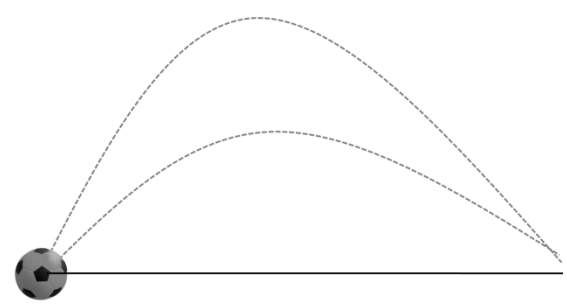
$R_1 = R_2$

$T_1 \neq T_2$

$H_1 \neq H_2$

In case of same range relation of time of flights

Here we know $\alpha + \beta = 90°$



Ball 1

$T_1 = \frac{2u\text{Sin}\alpha}{g}$

Ball 2

$T_2 = \frac{2u\text{Sin}\beta}{g}$ but $\beta = 90 - \alpha$

$= \frac{2u\text{Sin}(90-\alpha)}{g}$

$= \frac{2u\text{Cos}\alpha}{g}$

Now

$T_1 \times T_2 = \frac{2u\text{Sin}a}{g} \times \frac{2u\text{Cos}\alpha}{g}$

$\text{Sin}2\alpha = 2\text{Sin}\alpha\text{Cos}\alpha$

$T_1 \times T_2 = \frac{2}{g}\frac{u^2 Sin2\alpha}{g}$

$T_1 \times T_2 = \frac{2}{g} R_{same}$

In case of same range relation of maximum heights

Here again $\alpha + \beta = 90°$

Ball 1

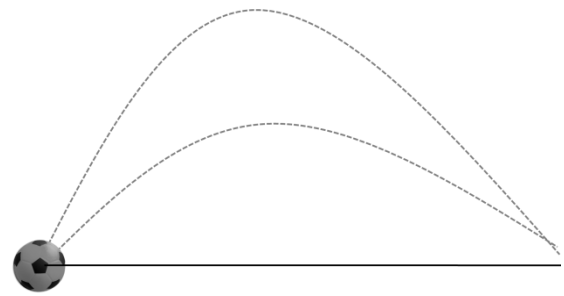
$H_1 = \frac{u^2 \sin^2 \alpha}{2g}$

Ball 2

$H_2 = \frac{u^2 \sin^2 \beta}{2g}, \beta = (90 - \alpha)$

$= \frac{u^2 \sin^2(90-\alpha)}{2g}$

$= \frac{u^2 \cos^2 \alpha}{2g}$



$H_1 \times H_2 = \frac{u^2 Sin^2\alpha}{2g} \times \frac{u^2 Cos^2\alpha}{2g}$

$= \frac{u^2}{2g} \times \frac{4u^2 Sin^2\alpha Cos^2\alpha}{2g \times 4}$

$= \left(\frac{u^2 Sin2\alpha}{g}\right)^2 \times \frac{1}{16}$

$H_1 \times H_2 = \frac{R^2}{16}$

$R = 4\sqrt{H_1 H_2}$