Chapter 3

Vectors and Scalars

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VECTOR AND SCALARS

Physical Quantity

May have numerical value, units and any specified direction.

Physical quantity = n u + specified direction

n u → Must have

specified direction → May have

Magnitude of any quantity = n u

n → Numerical Value

u → Units

Note:- Some physical quantities only having numerical value not specified direction.

Ex:- Refractive index, strain etc.

We can say any physical quantity Must/may have numerical value(n), unit (u) and specified direction.

PHYSICAL QUANTITY (N + U + DIRECTION) MAY/MUST HAVE

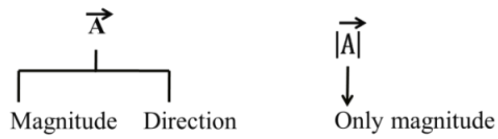
Scalars	Vectors	Tensors
"This type of physical quantities never required any specified direction" "But it may have direction Ex: Electric current (Scalar quantity) 10 Å Ex:-Electric current (scalar quantity) (n + u direction) " on addition, subtraction and product we do not need to follow any specified rules or Laws" Example: Distance, speed, time mass volume, All types of energies work done, Electric or magnetic flux, temperature etc.	" this types of physical quantities must have/ required any specified direction" " on addition, subtraction and product we must have to follow certain laws or rules" Ex:- For addition or subtraction is law of parallelogram. For product = dot or cross products Example:- Displacement, velocity All types of fields intensities, force torque, momentum etc.	" This type of physical quantities have no specified direction but different values in different direction" Ex:- Moment of inertia refractive index stress, strain etc.

Note:

Scalars or zero order tensors: Any physical quantity have only one component.

Vectors or first order tensors: Any physical Quantity have component greater then one but less than or equal to four.

REPRESENTATION OF VECTOR:



Types of vectors

(1) Polar Vectors:

Vectors related to linear motion of any object Ex. Displacement, velocity, Force etc.

(2) Axial vectors:

Vector represent rotational effect and are always along the axis of rotation Ex: Angular velocity, torque, angular momentum etc.

(3) Null vector or zero vectors:

Vector whose magnitude is zero and direction in determinant.

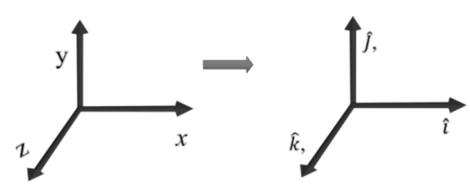
(4) Unit vectors:

Vector having magnitude equals to one (unity) but must be some specified direction representation of unit vector – , $\hat{A}$

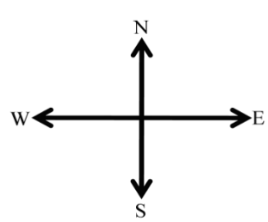
$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

STANDARD UNIT VECTOR

These are $\hat{\imath},\ \hat{\jmath},\ \hat{k},$



Direction should be on your copy

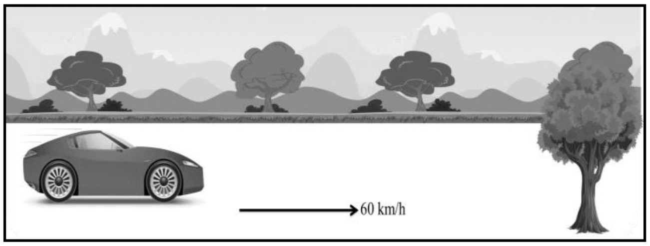


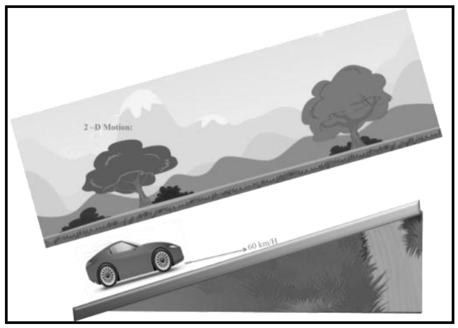
UNDERSTANDING OF UNIT VECTORS

1 –D Motion: Motion along *x* or *y* or *z*

A car is moving with 60 k/m toward or due east then speed (scalar) = 60 km/h

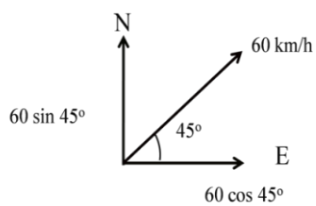
Velocity (Vector) = 60 km/h





2 –D Motion:

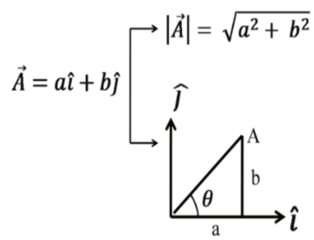
A car is moving with 60 k/m due north – east then



$\overrightarrow{V_{car}} = (60\cos 45°)i + (60\sin 45°)\hat{j}$

$= \left(\frac{60}{\sqrt{2}}\hat{\imath} + \frac{60}{\sqrt{2}}\hat{\jmath}\right) k/h$

2 –D vector



$\tan\theta' = \frac{b}{a} = \frac{\text{coeff.of } \hat{j}}{\text{coeff.of } \hat{i}}$

With x axis or with horizontal

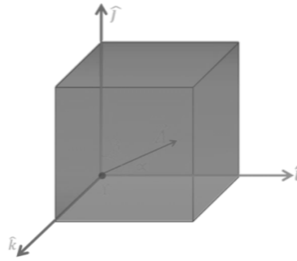
$\tan\theta = \frac{a}{b} = \frac{\text{coeff.of } \hat{\imath}}{\text{coeff.of } \hat{\jmath}}$

With Y-axis or with vertical

3 –D Vector

if $\hat{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$|A| = \sqrt{a^2 + b^2 + c^2}$



DIRECTION

$\cos \propto = \frac{a}{\sqrt{a^2+b^2+c^2}}$

With x-axis

$Cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}$

With y-axis

$Cosy = \frac{c}{\sqrt{a^2+b^2+c^2}}$

With z-axis

ADDITION AND SUBSTRACTION OF VECTORS

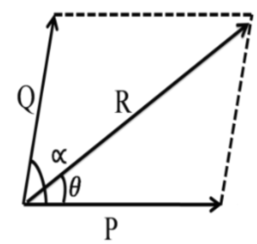
Law of parallelogram

Let $\vec{R} = \vec{P} \pm \vec{Q}$,

Here $\vec{P}$ and $\vec{Q}$ are two vector to be added or substracted and $\vec{R}$ is resultant vector.

Now magnitude of resultant $\vec{R}$

$R^2 = P^2 + Q^2 \pm 2PQ\ Cos\alpha$ here α is angle between $\vec{P}$ and $\vec{Q}$



For $R_{max} \rightarrow \alpha = 0° \ \vec{P} \uparrow\uparrow \vec{Q}$ (Parallel)

Then $R_{max} = P + Q$

For $R_{mix} \rightarrow \alpha = 180°, \vec{P} \downarrow\uparrow Q$ (Anti parallel)

Then $R_{min} = P \sim Q$

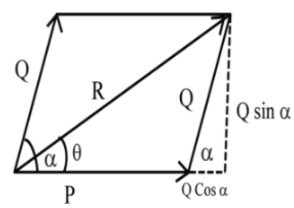
Similarly if $\alpha = 90°, P \perp Q$ (Perpendicular)

$R = \sqrt{P^2 + Q^2}$

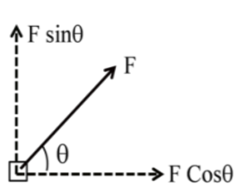
Direction of resultant vector $\vec{R}$

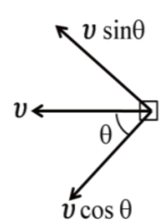
Angle of $\vec{R}$ with horizontal (or with $\vec{p}$)

$\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}$



RESOLVING OF ANY VECTOR





Example. Two equal vector have a resultant equal to either of the two. The angle between them is

(a) 90^o (b) 60^o

(c) 120^o (d) 0^o

Solution: By using expression $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

Let $|R| = |P||Q| = x$ then $x^2 + x^2 + 2x\,x\cos\alpha$

$x^2 = 2x^2(1 + \cos\alpha)$

$\frac{1}{2} = 1 + Cos\alpha$

$\cos\alpha = -1/2 = \alpha = 120^o$ **Answer is (c)**

Example: Two vector having equal magnitude of x units acting at an angle of 45^0 have resultant $\sqrt{(2) + \sqrt{2})}$ units the value of x is

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

Solution: Using the expression $R^2 = P^2 + Q^2 + 2PQ\cos\propto$

Here $|\vec{P}| = |\vec{Q}| = x, |\vec{R}| = \sqrt{2 + \sqrt{2},} x = 45$

$\left(\sqrt{2 + \sqrt{2}}\right)^2 = x^2 + x^2 + 2x\,x\cos 45°$

$(2 + \sqrt{2}) = 2x^2 + 2x^2\cos 45°$

$(2 + \sqrt{2}) = 2x^2 + \sqrt{2}\,x^2$

$(2 + \sqrt{2}) = x^2(2 + \sqrt{2})$

$\Rightarrow x^2 = 1$ **Ans (b) $\Rightarrow$ x = 1 Ans (b)**

Example: If $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$ find the angle between $\vec{P}$ and $\vec{Q}$

(a) 0° (b) 180°

(c) 90° (d) 120°

Solution: As given $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$

$\sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$

$= 4PQ\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^o$ **Answer is (c)**