Chapter 2 Errors

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ERRORS

The result of every measurement by any measuring instrument contains some uncertainty this uncertainty is called errors. The errors in a measurement is equal to the difference between the true value and the measured value of any quantity.

Errors = True value – Measured value

Absolute Errors:- the several values obtained in an experiment measured are a_1, a_2, a_3….an the arthmetic mean of these values will be

$$a_{mean} = \frac{(a_1+a_2+a_3+\cdots a_n)}{n}$$

Now error in first measurement = $\Delta a_1 = a_{mean} - a_1$

Similarly in second measurement = $\Delta a_2 = a_{mean} - a_2$ and so on

now mean absolute error $\Rightarrow \Delta a_{mean} = \frac{|\Delta a_1|+|\Delta a_2||\Delta a_3|+\cdots+|\Delta a_n|}{n}$

Relative error: Relative error $= \frac{\Delta a_{mean}}{a_{mean}}$

Percentage error: $\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$

Combination of errors (maximum possible error) **Addition or subtraction**

Let $C = a \pm b$

$$\frac{\Delta C}{C} = \frac{\Delta a}{a \pm b} + \frac{\Delta b}{a \pm b}$$

Product or multiplication

Let

$$C = a \times b$$

$$\frac{\Delta C}{C} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Division

Let

$$C = \frac{a}{b}$$

$$\frac{\Delta C}{C} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

(4) Powers

Let $C = a^n b^m$

$$\frac{\Delta C}{C} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b}$$

CONCEPT WITH EXAMPLES

Role of constants

The radius of sphere is measured to be (2.1 ± 0.5) cm and is surface area with error limits.

Solution:

Surface area = A = $4\pi r^2$

$\frac{\Delta A}{A} = \frac{2\Delta r}{r}$

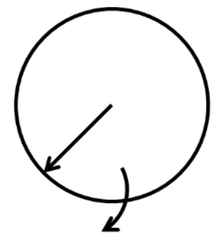
$= 2 \times \frac{0.5}{2.1}$

$\frac{\Delta A}{A} = \frac{10}{21}$

Now% error

$\frac{\Delta A}{A} \times 100 = \frac{10}{21} \times 100$

= 47.62 %



r = (2.1 ± 0.5) cm

Other way

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (2.1)^2$

= 55.4 cm^2

Now $= \frac{\Delta A}{A} = 2\frac{\Delta r}{r}$

$\Delta A = 2\frac{\Delta r}{r} \times A$

$= 2 \times \frac{0.5}{2.1} \times 55.4$

= 26.4 cm^2

Now $(A \pm \Delta A)$

$= (55.4 \pm 26.4)$

FINDING QUANTITY

Percentage error in determining of acceleration of gravity with the help of simple Pendulum time period $T = 2\pi\sqrt{\frac{l}{g}}$. Given error in length of pendulum 4% while in The time period it is 2% then percentage error in g = ?

Solution

Wrong way

$T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} + \frac{1}{2}\frac{\Delta g}{g}$

$2\% = \frac{1}{2}\frac{\Delta l}{l} + \frac{1}{2}\frac{\Delta g}{g}$

$2\% = \frac{1}{2} \times 4\% + \frac{1}{2}\frac{\Delta g}{g}$

Correct way

$T = 2\pi\sqrt{\frac{l}{g}}$

$T^2 = 4\pi^2\frac{l}{g}$

$g = 4\pi^2\frac{l}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$

$= 4\% + 2 \times 2\%$

$\frac{\Delta g}{g} = 8\%$

LEAST COUNT AND DIFFERENT UNITS

If a particle of mass m = 25.0 Kg is moving in a circular path of radius r = 50.00 cm with constant speed of v = 10.0 Km/hr them find error in force required by particle.

Now Given

m = 25.0 Kg

= L.c. = 0.1

V = 10.0 Km/h

= L.c = 0.1

ΔV = 0.1, V = 10

R = 50.00 cm

Δr = 0.001, r = 50cm

NOTE: never change units in error.

Least count

25.00 – L.c = 0.01

16.50 – 50 –L.c = 0.01

76.03 – 03 –L.c = 0.01

15.0002 –L.c = 0.0001

17.030 – L.c = 0.01

Solution:

$F = \frac{mv^2}{r}$

$\frac{\Delta F}{F} = \frac{\Delta m}{m} + \frac{2\Delta v}{V} + \frac{\Delta r}{r}$

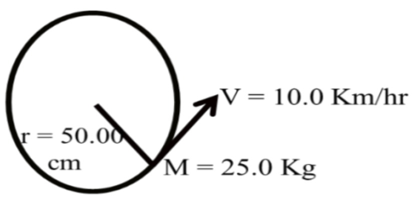
$\frac{\Delta F}{F} = \frac{0.1}{25} + \frac{2\times 0.1}{10} + \frac{0.01}{50}$

$= \frac{0.2+1+0.01}{50}$

$\frac{\Delta F}{F} = \frac{1.21}{50}$

In% $\frac{\Delta F}{F} = \frac{1.21}{50} \times 100$

= 1.21 x 2= 2.42% **Ans.**



UNIQUE CONCEPT

Calculate focal length of a spherical Mirror from the following observations Object distance u = (50.1 ± 0.5) cm. and image distance v = (20.1 ± 0.2) cm.

Solution: $-\frac{1}{f} = \frac{1}{V} + \frac{1}{u}$

$\frac{1}{f} = \frac{u+V}{Vu} = f = \frac{Vu}{V+u}$

$= \frac{50.1\times 20.1}{50.1+20.1}$

= 14.3 cm

Now

$f = \frac{Vu}{V+u}$

$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u} + \frac{\Delta V}{V} + \frac{\Delta u+\Delta v}{u+v}\right]$

$= \pm\left[\frac{0.5}{50.1} + \frac{0.2}{20.1}\right] + \frac{0.5+0.2}{50.1+20.1}$

$\frac{\Delta f}{f} = \pm[0.0299]$

$\Delta F = \pm[0.0299] \times 14.3$

± 0.4 cm

Note: Similar case in resistance connected in parallel combinations

$$\frac{1}{Req.} = \frac{1}{R_1} + \frac{1}{R_2}$$

SIGNIFICANT FIGURES

The number of significant figures of a numerical quantity is the number of reliably known digits it contains:

Rules:

1. Zeros at the beginning of a number are not significant.
Ex. 0.0523 – there S.f. (5, 2, 3)

2. Zeros within a number are significant.
Ex: 2056 –Four S.f. (2, 0, 5, 6)

3. Zeros at the end of a number after decimal points are significant.
Ex. 3702.0 –five S.f. (3, 7, 0, 2, 0)