Chapter 1 Unit and Dimensions

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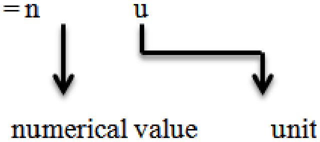
UNITS AND DIMENSIONS

Units “ To measure or represent any physical quantity we need units”

Exception: Although there are few physical quantities which do not need any unit like refractive index, Relative density or specific gravity etc.

Physical quantity = n u



nu = Constant

$n \propto \frac{1}{u}, n_1 u_1 = n_2 u_2$

Example: In a particular system, the unit of length, mass and time are chosen to be 10cm, 10gm and 0.1 sec respectively. The unit of force in this system will be equivalen to:

(a) 0.1 N

(b) 1 N

(c) 10 N

(d) 100 N

Sol. Unit of force ⇒ F = ma = Kg metre /sec^2

Here $F = \frac{(10\text{gm})(10\text{cm})}{(0.1\,\text{sec})^2} = \frac{(10\times10^{-3}\text{kg})(10\times10^{-2}\text{metre})}{(10^{-1}\,\text{sec})^2}$

F = 0.1 N

So the Correct option is (a)

Example: The density of a material in CGS System of unit is 4gm/cm^3. In a system of units in which unit of length is 10cm and unit of mass is 100 gm the value of density of material will be

(a) 0.04

(b) 0.4

(c) 40

(d) 400

Solution. Given $n_1 u_1$ = 4 gm/cm^3

$n_2 u_2 = n_1 u_1$

$n_2u_2 = 4\frac{gm}{cm^3} \Rightarrow n_2 x \frac{100gm}{(10cm)^3} = \frac{4gm}{cm^3}$

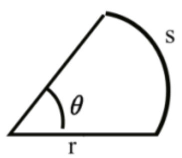
$n_2 = 40$ **Answer (a)**

FUNDAMENTAL PHYSICAL QUANTITIES, UNITS, SYMBOLS

S.No	Physical quantity	Unit	Symbol
1.	Mass	Kg	M
2.	Length	Meter	L
3.	Time	Second	T
4.	Temperature	C°	θ ,or K
5.	Electric Current	Ampere	A or I
6.	Luminous Intensity	Candela	Cd
7.	Amount of Substance	Mole	Mol or m

SUPPLEMENTARY UNITS

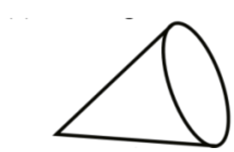
(1) Angle



$\text{angle}(\theta) = \frac{\text{arc}}{\text{radius}}$, unit; Radian

$\theta = \frac{s}{r}\ \pi\ radian = 180°$

(2) Solid angle



Unit = St. Radian

$\Omega = \phi = \frac{A}{r^2}$

Derived Units: The units which may derive by fundamental or Supplementry units

Ex. Force= ma = Kg mt/sec^2

Conventions adopted while writing a unit

1. Even if a unit is named after a person it should not be Capital initial letter.

Newton (no) Joule (no) Ampere (no)

or or etc.

newton (yes) joule (yes) ampere (yes)

2. In symbol for a unit named after a person

Newton $\rightarrow$ N

Ampere $\rightarrow$ A

Joule $\rightarrow$ J

Watt $\rightarrow$ w

3. The symbol or units are not expressed in plural form : 50 m (yes) , 10 joule (yes)
50 ms (no) , 10 joules (no)
4. Not more than one Solidus is used 1 poise = 1g/s cm or 1 gs^{-1} cm^{-1} not 1 g/s /cm
5. Full stops are not written after abbreviations and units 1 litere = 1000 c c (yes)
= 1000 c.c. (No)

SYSTEMS OF UNITS

(1) MKS AND MKSA SYSTEM

Here in this system M→ meter, K→ Kilogram, S →Second and A→ ampere

Physical quantity Unit in MKS or MKSA System

(1)Force $\xrightarrow[F=ma]{}$ kg met/sec^2

(2)Energy $\xrightarrow[\frac{1}{2}mv^2=mgh]{}$ kg met^2/sec^2

(3)Power $\xrightarrow[\frac{1}{2mv^2}/time]{}$ kg met^2/sec^3

…….. and so on

(2) SI SYSTEM (INTERNATIONAL SYSTEM)

Here in this system unit of physical quantities are use to be named after different great Scientists.

Physical quantity

Unit in SI System

1. Force → newton or N
2. Energy → joule or J
3. Power → watt or W

(3) CGS SYSTEM

Here C→ Centimeter, G → gram, S → Second

Physical quantity Unit in CGS System

(1)Force $\xrightarrow[F=ma]{}$ gcm/sec^{2}ordyne $1N = 10^5$dyne

(2)Energy $\xrightarrow[\frac{1}{2}mv^2=mgh]{}$ gcm^2/sec^{2}or erg $1J = 10^7$erg

(3)Power → gcm^2/sec^3

….and so on

(4) FPS SYSTEM

Here F → foot, P → pound , S → second

Force → pd ft/ sec^2, Similarly other physical quantities

Dimensional and dimensional formula

Dimensional formula

Almost all physical quantities can be expressed in terms of the seven fundamental units in symbolic form.

Dimensions

Dimensions of a physical quantity are the powers to which we must be raised to fundamental quantity represent the given physical quantity.

	Physical quantity	Dimensional formula	Dimension
1. Force ,	F = ma	$[MLT^{-2}]$	[1,1, -2]
2. Universal gravitational Constant $F = G\frac{m_1 m_2}{r^2}$		$[M^{-1} L^{3} T^{-2}]$	[-1, 3, -2]
3. Planks Constant $E = \frac{hc}{\lambda}$		$[ML^2T^{-1}]$	[1, 2, -1]
4. Boltz man Constant $E = \frac{3}{2}KT$		$ML^2T^{-2}\theta^{-1}]$	[1,2, -2, -1]
5. Universal gas Constant		$[ML^2T^{-2}\theta^{-1} mol^{-1}]$	[1, 2, -2, -1, -1]

……. And so on

APPLICATIONS OF DIMENSIONAL FORMULAS

Application No. (1)

To establish results

Suppose any mass m is moving along a circular path of radius R with uniform speed of V. If the force required to do so is F then establish the result or relation between F, m, v and R.

Given ,

$F = k\ m^x v^Y R^Z$

$F \alpha\ m^x v^Y R^Z$

$[M\ L\ T^{-2}] = K\ [M]^x\ [LT^{-1}]^y\ [L]^z$

$[M\ LT^{-2}] = K\ [M^x L^{y+z} T^{-y}]$

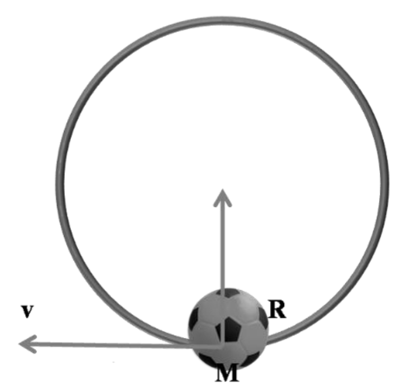
Comparing both sides

$X = 1,\ y + z = 1,\ -y = -2 \Rightarrow y = 2$

$2 + z = 1$

$Z = 1 - 2 = -1$

By experiment Constant $K = 1$



So we have result $F = m^1 v^2 R^{-1} \Rightarrow F = \frac{mv^2}{R}$

Example 1 Given a simple pendulum of very small bob of mass connected with a light String of length *l*. Also given acceleration due to gravity is g and time period *T* of simple pendulum depends on mass of the bomb, length of the String *l* and acceleration due to gravity g then establish result for T.

Given

$T \propto m^x l^y g^z$

$T = k m^x l^y g^z$

$[M^\circ L^\circ T^1] = K [M]^x [L]^y [LT^{-2}]^z$

$\Rightarrow [M^\circ L^\circ T^1] = k [M^x L^{y+z} T^{-2z}]$

Compare both Side

$x = 0$, *i.e*. Time period will not depend on mass of the bob $Y + z = 0$

Compare both Side

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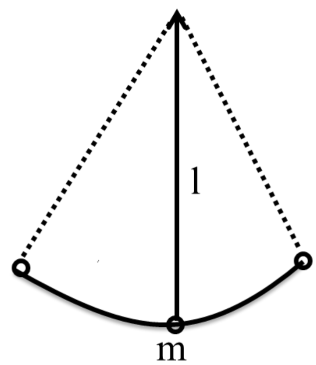


$Y = -z \Rightarrow Y = -\left(-\frac{1}{2}\right) = \frac{1}{2} - 2Z = 1$

$Z = -\frac{1}{2} t = \left[km^\circ l^{1/2} g^{-1/2}\right]$

By experiment k = 2π

$$T = 2\pi\, l^{1/2} g^{-1/2} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

Example: In a system of units , if force F, acceleration A and time T are taken as fundamental units then the dimensional formula of energy is

(a) $[F A^2 T]$

(b) $[F A T^2]$

(c) $[F^2 AT]$

(d) [FAT]

Solution: Let energy denote by E

Then $E \propto F^x A^y T^z$

$E = F^x A^y T^z$

$[M L^2 T^{-2}] = [M L T^{-2}]^x [LT^{-2}]^y [T]^Z$

$[M L^2 T^{-2}] = [M^x L^{x+y} T^{-2x-2y+z}]$

Compare both Sides

x =1 , - 2x – 2y + z = -2

x + y = 2 - 2x1 -2 x1 + z = -2

⇒ Y = 1 - 4 + z = -2

Z = 4 – 2

⇒ Z = 2

Therefore dimensional formula of energy E in Terms of FAT is $[F^1 A^1 T^2] \Rightarrow [FAT^2]$ **Ans B**

Application No.2: To check the validity of any result.

Law of homogeneity

Here we can jump through +, - , = (signs)

If $v = at + \frac{b}{t+c}$ (Here v → velocity t → time

and a,b, c- are Constant then find dimensional formula of a, b, c-

Given: $v = at + \frac{b}{t+c}$

Apply Law of homoginity

Dimensional formula of v = dim-formula of at = v

$a = \frac{v}{t} = \frac{[LT^{-1}]}{[T]} = [LT^{-2}]$

Similarly we can say t = c ⇒ C = [T]

and $v = \frac{b}{t} \Rightarrow b = vt = [LT^{-1}][T]$

b = [L]

Example: The equation of state of some gases can be expressed as $\left(p + \frac{q}{v^2}\right)(v - b) = RT$ here p is pressure, V is the volume, T is temp. then the dimensional formula of Constants a and b will be ?

Given: $\left(p + \frac{a}{V^2}\right)(V - b) = RT$

Apply Law of homoginity Dimensional formula of p = Dim-formula of $\frac{a}{V^2}$

$a = PV^2 = ML^{-1}T^{-2}L^6 = [ML^5T^{-2}]$

Similarly dimensional formula of v = Dim. Formula of b b = v

$= [L^3]$

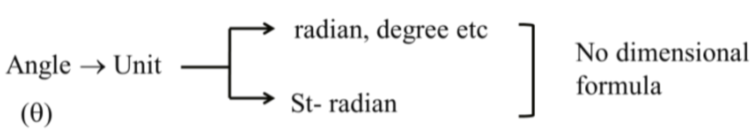
ANS $a = [ML^5 T^{-2}]$

$b = [L^3]$

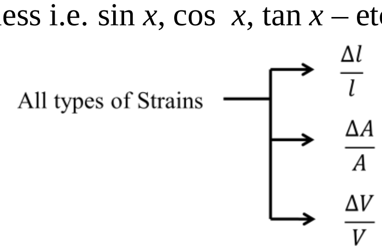
Application No-3

Dimension less quantities or functions

(i) Quantities having units but dimensionless



So all trigonometric functions will be dimension less i.e. sin *x*, cos *x*, tan *x* – etc.



(ii) Quantities neither having units nor dimension

Refraction index $n = \frac{c}{v}$

Reynolds number, Relative density etc.

(iii) Dimension less mathematical functions

(a) Logarithmic functions: $log_e\ x$ or $log_{10}\ x$

(b) Exponential functions: e^x, a^x

Exmaple. If $p = \frac{RT}{V-b} e^{-\alpha V/RT}$, then dimensional formula of α is

(a) P

(b) R

(c) T

(d) V

Solution: Given $p = \frac{RT}{V-b} e^{-\alpha v/RT}$

Here $^{-\alpha v/RT}$ will be dimension less

So $\alpha\ v = RT$

$\alpha = \frac{RT}{V}$ now for 1 mole of gas $PV = nRT$

$\alpha = \frac{PV}{V} = P$ **Correct answer is (a)**

Example Nuclear force between nucleons is given by $F = C\frac{e^{-kr}}{r^2}$ Here F is force, r is distance, and c, k are Constants then find dimensional formula of C and k = ?

Solution: Given: $F = C\frac{e^{-kr}}{r^2}$

Here e^{-kr} is dimension less

So $C = Fr^2 = M\ L\ T^{-2}\ L^2 = [M\ L^3\ T^{-2}]$

No because e^{-kr} is dimension less

So k = 1/r

= $[L^{-1}]$

Ans: dimensional formula of

$C = [M\ L^3 T^{-2}]$

$K = [L^{-1}]$

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