Chapter

Basic Physics

Day - 1

DIFFERENTIATION

"It allows us to find rate of change"

"Rate of change of any physical Quantity with respect to any other quantity & Finding the differentiation result of a function with respect to x, y, z means Finding slope to the x- axis from the curve"

Symbol; $\frac{dy}{dx}$ or $\frac{dx}{dt}$ or $\frac{dv}{dt}$ etc.

$\frac{d}{dx}x^n = nx^{n-1}$ Here n is any number, fraction positive or negative

Ex. $\frac{d}{dt}t^5 = 5t^4$

$\frac{d}{dx}Nx^n = Nnx^{n-1}$ Here N and n both are any number or fraction.

Ex. $\frac{d}{dt}7t^3 = 21\,t^2$

$\frac{d}{dt}\frac{x^n}{N} = \frac{1}{N}nx^{n-1}$

Ex. $\frac{d}{dt}\frac{t^6}{10} = \frac{6}{10}t^5$

$\frac{d}{dx}(x^n \pm N) = \frac{d}{dx}x^n \pm \frac{d}{dx}N = nx^{n-1}$

Ex. $\frac{d}{dt}(t^2 \pm 7) = 2t, \frac{d}{dx}$ constant $= 0$

APPLICATIONS

(1) **Velocity**: Rate of change of displacement. $v = \frac{dx}{dt}$ or $\frac{dy}{dt}$ or $\frac{dx}{dt}$ or $\frac{dx}{dt}$

(2) **Acceleration**: Rate of change of velocity $\left(a = \frac{dv}{dt}\right)$ and $a = v\frac{dv}{ds}$

(3) **Force**: Rate of change of linear momentum $F = \frac{dp}{dt}$

(4) **Torque**: Rate of change of angular momentum $\tau = \frac{dL}{dt}$

(5) **Induce EMF**: "Rate of change of magnetic flux $E = -\frac{d\phi}{dt}$ (for single turn coil)

(6) Potential gradient (Intensity of electric field) $E = \frac{dV}{dr}$ and so on many more

Example: if position of any moving particle is changing as **x =** $(t^3 - 2t^2 + t - 1)$ then find

(i) Initial velocity of the particle.

(ii) Initial acceleration of the particle.

(iii) At t = 2 sec, velocity of the particle

(iv) At t = 1 sec, acceleration of the particle

(v) position of the particle when it is moving with constant velocity

(vi) Maximum velocity of the particle

Solution: Given $x = (t^3 - 2t^2 + t - 1)$

Now $V = \frac{dx}{dt} = \frac{d}{dt}(t^3 - 2t^2 + t - 1) = 3t^2 - 4t + 1$

$V = 3t^2 - 4t + 1$

Now, $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 4t + 1)$

$= 6t - 4$

(1) Initial velocity ≡ Means velocity at t = 0

So $V = 3(0)^2 - 4 \times 0 + 1$

$V = 1$ m/s

(ii) Initial acceleration ≡ Means acceleration at t = 0

So $a = 6t - 4$

$a = 6(0) - 4$

$= -4$ m/s^2 (Retardation)

(iii) velocity at t = 2 sec

$v = 3t^2 - 4t + 1$

$= 3(2)^2 - 4 \times 2 + 1$

$= 12 - 8 + 1 = 5$ m/s

(iv) Acceleration at t = 1 sec

$a = 6t - 4$

$= 6 \times 1 - 4 = 2$ m/s^2

(v) Position when moving with constant velocity (a = 0)

$a = 6t - 4 = 0$

$6t = 4$

$t = \frac{4}{6} = \frac{2}{3}$ sec

Now $x = t^3 - 2t^2 + t - 1$

$= \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + \frac{2}{3} - 1$

$= \frac{8}{27} - \frac{8}{9} + \frac{2}{3} - 1$

$= \frac{8-24+18-27}{27} = \frac{-25}{27}$ met

(vi) Maximum velocity = Means a = 0

$a = 6t - 4 = 0$

$t = \frac{2}{3}$ sec

MORE DETAILS ON DIFFERENTIATION

$\frac{d}{dx}(x + N)^n = n(x + N)^{n-1}$

Ex. $\frac{d}{dt}(t + 5)^3 = 3(t + 5)^2$

$\frac{d}{dx}(x^m + N)^n = n(x^m + N)^{n-1}\, mx^{m-1}$

Ex. $\frac{d}{dt}(t^3 + 7)^4 = 4(t^3 + 7)^3 . 3t^2$

DOUBLE FUNCTIONAL DIFFERENTIATION

$\frac{d}{dx} x_1 \times x_2 = x_1 \frac{d}{dx} x_2 + x_2 \frac{d}{dx} x_1$

Ex. $\frac{d}{dt}t^3(t+5)^2 = t^3\frac{d}{dt}(t+5)^2 + (t+5)^2\frac{d}{dt}t^3$
$= t^3 \times 2(t+5) + (t+5)^2 \times 3t^2$

$$\frac{d}{dx}\frac{x_1}{x_2} = \frac{x_2\frac{d}{dx}x_1 - x_1\frac{d}{dx}x_2}{x_2^2}$$

Ex. $\frac{d}{dt}\frac{t^2}{(t+5)^2} = \frac{(t+5)^2\frac{d}{dt}t^2 - t^2\frac{d}{dt}(t+5)^2}{(t+5)^4} = \frac{(t+5)^2 \times 2t - t^2 \times 2(t+5)}{(t+5)^4}$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

(1) $\frac{d}{dt}\sin t = \cos t$

(2) $\frac{d}{dx}\cos x = -\sin x$

(3) $\frac{d}{dx}\tan x = \sec^2 x$

(4) $\frac{d}{dx}\cot x = -\text{cosec}^2 x$

(5) $\frac{d}{dx}\sec x = \sec x \tan x$

(6) $\frac{d}{dx}\text{cosec}\, x = -\text{cosec}\, x \cot x$

Example: velocity of a moving particle is given by v=10 sin t. Then find its acceleration $t = \frac{\pi}{6}$ sec,

Solution: Given V = 10 sin t

$a = \frac{dv}{dt} = \frac{d}{dt}10\sin t$
$= 10 \cos t$

$10\cos\frac{\pi}{6} - 10\cos 30$

$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{m/s}^2$

DIFFERENTIATION OF EXPONENTIAL AND LOGARITHMIC FUNCTION

(1) $\frac{d}{dx}\log_a x = x = \frac{1}{x}$ or $\frac{d}{dx}$ in $x = \frac{1}{x}$

(2) $\frac{d}{dx}e^x = e^x$

Example: position of a moving particle is given by 20 e^t then its initial velocity will be?

Solution: $X = 20\,e^t$ (given)

$$V = \frac{dx}{dt} = \frac{d}{dt}20e^t = 20e^t$$

Put t = 0

V = 20 m/s **Answer**

Example: A particle is moving with speed $v = b\sqrt{x}$ along positive X- axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0)

(2019 JEE Mains)

$(a)\ \frac{b^2\tau}{4}$ $(b)\ \frac{b^2\tau}{4}$

$(c)\ b^2\tau$ $(d)\ \frac{b^2\tau}{\sqrt{2}}$

Solution: Given, speed

$v = b\sqrt{x}$

Now, differentiating it with respect to time, we get

$\frac{dv}{dt} = \frac{d}{dt}\ b\sqrt{x}$

Now, acceleration

$\Rightarrow \quad a = \frac{b}{2\sqrt{x}}.\frac{dx}{dt} \qquad \left[\because \frac{dv}{dt} = a\right]$

$\Rightarrow \ a = \frac{b}{2\sqrt{x}}.v = \frac{b}{2\sqrt{x}}.b\sqrt{x} = \frac{b^2}{2}$

As acceleration is constant, we use

$v = u + at$ …….(i)

Now, it is given that x = 0 at t =0.

So, initial speed of particle is

$u = b\sqrt{x} \quad |x = 0 = b \times 0 = 0$

Hence, when time $t = \tau$, speed of the particle using Eq. (i) is

$v = u + at = 0 + \frac{b^2}{2}\ .\ \tau = \frac{b^2}{2}\ .\tau$ **Answer (b)**

Example: The position vector r of particle of mass m is given by the following equation $\vec{r}\,(t) = \alpha t^3\hat{\imath} + \beta t^2\,\hat{\jmath}$ where, $\alpha = \frac{10}{3}\ ms^{-3}$, $\beta = 5ms^{-2}$ and m =0.1 kg. At t =1s, which of the following statements is (are) true about the particle? **(2016 JEE Adv.)**

(a) the velocity v is given by $v = (10\hat{\imath} + 10\hat{\jmath})ms^{-1}$

(b) The angular momentum L with respect to the origin is given by $L = \left(\frac{5}{3}\right)\hat{k}\ Nms$

(c) The force F is given by $F = (\hat{\imath} + 2\hat{\jmath})N$

(d) The torque τ with respect to the origin is given by $\tau = -\frac{20}{3}\hat{k}\ Nm$

Solution: $r = \alpha t^3\,\hat{\imath} + \beta t^2\hat{\jmath}$

$v = \frac{dr}{dt} = 3\alpha t^2\hat{\imath} + 2\beta t\hat{\jmath}$

$a = \frac{d^2r}{dt^2} = 6\alpha t\,\hat{\imath} + 2\beta\hat{\jmath}$

At t =1s,

(a) $v = 3 \times \frac{10}{3} \times 1\hat{\imath} + 2 \times 5 \times 1\hat{\jmath} = (10\hat{\imath} + 10\hat{\jmath})m/s$

(b) $L = r \times p = \left(\frac{10}{3} \times 1\hat{\imath} + 5 \times 1\hat{\jmath}\right) \times 0.1\ (10\hat{\imath} + 10\,\hat{\jmath}\,)N\ ms$

$= \left(-\frac{5}{3}\hat{k}\right)N - ms$

(c) $F = ma = m\ \times F = \left(6 \times \frac{10}{3} \times 1\hat{\imath} + 2 \times 5\hat{\jmath}\right) \times (2\hat{\imath} + \hat{\jmath})N$

(d) $\tau = r \times F = \left(\frac{10}{3}\hat{\imath} + 5\hat{\jmath}\right) \times (2\hat{\imath} + \hat{\jmath})$

$= \ +\frac{10}{3}\ \hat{k} + 10(-\hat{k}) = \left(-\frac{20}{3}\ \hat{k}\right)N - m$ **Answer (a, d)**

Example: The position vector of particle change with time according to the relation $r(t) = 15t^2\hat{\imath} + (4 - 20t^2)\hat{\jmath}$. What is the magnitude of the acceleration (in ms^{-2}) at t =1?

(a) 50 (b) 100
(c) 25 (d) 40

Solution: Positon vector of particle ids given as

$r = 15\, t^2\hat{\imath} + (4 - 20t^2)\hat{\jmath}$

Velocity of p[article is

$v = \frac{dr}{dt} = \frac{d}{dt} \quad [15t^2\,\hat{\imath} + (4 - 20t^2)\hat{\jmath}]$

$= 30t\,\hat{\imath} - 40\,t\,\hat{\jmath}$

Acceleration of particle is

$a = \frac{d}{dt}(v) = \frac{d}{dt}(30t\hat{\imath} - 40t\hat{\jmath}) = 30\hat{\imath} - 40\hat{\jmath}$

So, magnitude of acceleration at t =1s is

$|a|_{t=1s} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2}$

= 50 ms^{-2} **Answer (a)**

Example: The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be **(2019 JEE Mains)**

$(a)\ a + \frac{b^2}{2c}$ $(b)\ a = +\frac{b^2}{4c}$

$(c)\ a + \frac{b^2}{3c}$ $(d)\ a + \frac{b^2}{c}$

Solution: Position of particle is, x (t) = at + bt^2 –ct^3

So, its velocity is, $v = \frac{dx}{dt} = a + 2bt - 3ct^2$

And acceleration is, $a = \frac{dv}{dt} = 2b - 6ct$

Acceleration is zero, then $2b - 6ct = 0$

$\Longrightarrow t = \frac{2b}{6c} = \frac{b}{3c}$

Substituting this ‘t’ in expression of velocity, we get

$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$ **Answer (c)**

Day – 2 Question Practice Online