Chapter 3 CURRENT ELECTRICITY

Day-1

1. Current Resistance and Electromotive Force

1.1 Introduction

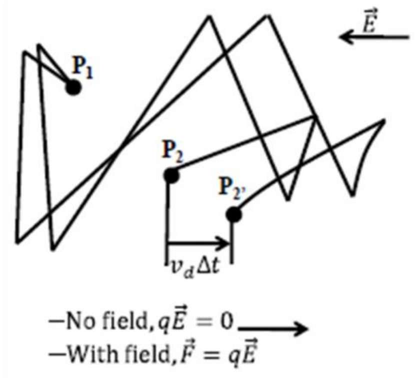
In the past four chapter we studied the interactions of electric charges at rest; now we're ready to study charge in motion. An electric current consists of charges in motion from one region to another. When this motion takes place within a conducting path that forms a closed loop, the path is called an electric circuit. We'll study the properties of batteries and how they cause current and energy, potential difference, resistance, and electromotive force. Finally, we'll look at electric current in a material from a microscopic viewpoint.

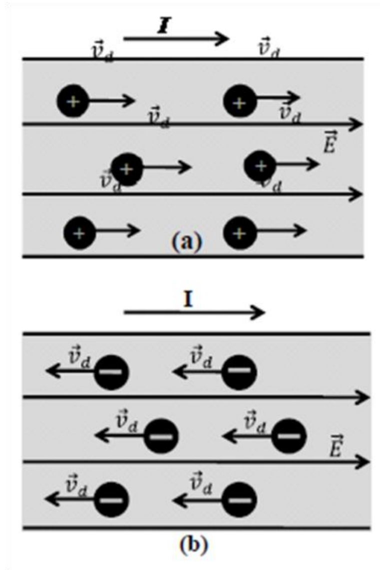
1.2 Current

A current is any motion of charge from one region to another. In this section we'll discuss currents in conducting materials. The vast majority of technological applications of chares in motion involve currents of this kind. In an ordinary metal such as copper or aluminum, some of the electrons move randomly in all directions, somewhat like the molecules of gas but with much greater speeds, of the order of 10^6 m/s. The electrons nonetheless do to not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no net flow of charge in any direction and hence no current. The drift of moving charges through a conductor can be interpreted in terms of work and energy. The electric field $\vec{E}$ does work on the moving charges. The resulting kinetic energy is transferred to the material of the conductor can be interpreted in terms of work conductor.

Shown in figure segments of two different current carrying materials. The moving charges are positive, the electric force is in the same direction as $\vec{E}$ and the drift velocity $\vec{v_d}$ is from right to left. In either case electric force is opposite to $\vec{E}$ and drift velocity $\vec{v_d}$ is from left and right, and positive charge end up to the right of negative ones. We define the current, denoted by I, to be in the direction in which there is a net flow of positive charge.

$$I = \frac{dQ}{dt} \qquad \text{(definition of current)}$$





1.3 Current, Drift Velocity and current Density

Suppose there are n charged particles per unit volume. We call n the concentration of particles; its SI unit is m^{-3}. Assume that all the particles move with the same drift velocity with magnitude v_d. In a time interval dt, each moves a distance v_d dt. The particles that flow out of the right end of the shaded cylinder with length v_d dt during dt are the particles that were within this cylinder at the beginning of the interval dt. The volume of the cylinder is Av_d dt, and the number of particles with in it is nAv_d dt. If each particles has a charge q, the charge dQ that flows out of the end cylinder during time dt is:

$$dQ = n(nAv_d\, dt) = nqv_dA\, dt$$

$$I = \frac{dQ}{dt} = nqv_dA$$

The current per unit cross-section area is called the current density J

$$J = \frac{I}{A} = nqv_d$$

$$I = \frac{dQ}{dt} = n|q|v_dA \qquad \text{(General expression for current)}$$

$$J = \frac{I}{A} = n|q|v_d \qquad \text{(General expression for current density)}$$

1.4 Resistivity

The current density $\vec{J}$ in a conductor depends on the electric field $\vec{E}$ and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, $\vec{J}$ is nearly directly proportional to $\vec{E}$ and the ratio of the magnitude E and J is constant. This relationship, called Ohm's Law was discovered in 1826 by German physicist Georg shimon Ohm. The word "Law" should actually be in quotes, since Ohm's law like the ideal gas equaquite well but is not a general description of all matter.

We define the resistivity ρ of a material as the ratio of the magnitude of electric filed and current density:

$$\rho = \frac{E}{J} \qquad \text{(Definition of Resistivity)}$$

RESISTIVITIES ROOM TEMPREATURE (20ºC)

	SUBTANCE	$\rho(\Omega.m)$	SUBTANCE	$\rho(\Omega.m)$
Conductors			Semiconductors	
Metals:	Silver	1.47×10^{-8}	Pure Carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure Silicon	2300
	Aluminum	2.75×10^{-8}	Illustration	
	Tungsten	5.25×10^{-8}	Amber	5×10^{-14}
	Steel	20×10^{-8}	Glass	$10^{10} - 10^{14}$
	Lead	22×10^{-8}	Lucite	$> 10^{13}$
	Mercury	95×10^{-8}	Mica	$10^{11} - 10^{15}$
Alloys:	Manganin (Cu 84%, Mn 12%, Ni 40%)	44×10^{-8}	Quartz (fuesd)	75×10^{-14}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$> 10^{13}$
			Wood	$10^{8} - 10^{11}$

1.5 Resistance

For a conductor with resistivity ρ, the current density $\vec{J}$ at a point where the electric field is $\vec{E}$ is given:

$$\vec{E} = \rho\vec{J}$$

When Ohm's Law is obeyed, ρ is constant and independent of the magnitude of the electric field so $\vec{E}$ is directly proportional to $\vec{J}$. Often however we are more interested in the total current in a conductor than in $\vec{J}$ and more interested in the potential difference between the difference are much easier to measure than are $\vec{J}$ and $\vec{E}$. Suppose our conductor is a wire with uniform cross – section area a and length L, as show in figure. Let V be the potential difference between the higher – potential and lower – potential ends of the conductor so that V is positive. The direction of the the current is always from the higher – potential end to the lower – potential end. That's because current in a conductor flows in the direction of $\vec{E}$ no matter what the sign of the moving charge and because $\vec{E}$ points in the direction of decreasing electric potential. As the current flows through the potential difference, electric potential difference energy is lost this energy is transferred to the ions of the conducting material during collisions.

$$\frac{V}{L} = \frac{\rho I}{A} \qquad \text{or} \qquad V = \frac{\rho L}{A} I$$

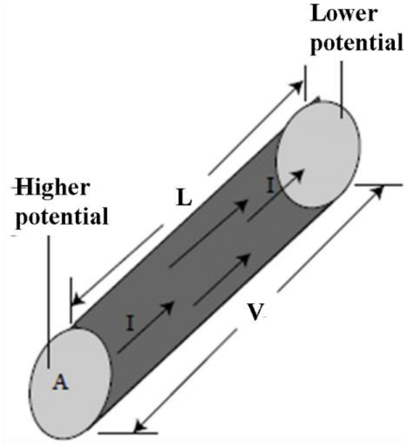
The ratio of V to I for a particular conductor is called its resistance R.

$$R = \frac{V}{I}$$

$$R = \frac{\rho L}{A} \qquad \text{(relationship between resistance and resistivity)}$$

If ρ is constant as is the case for ohmic materials, then so is R.

$V = IR$ (relationship between voltage, current and resistance)



A circuit device made to have a specific value of resistancebetween its ends is called a `resistor. Resistor in the range 0.01 to $10^7 \Omega$ can be bought off the shelf. Individual diameter and length, with wires coming out of the ends. The resistance may be marked with a standard code using three or four color bands near one end, according to the scheme. The first two bands are digits and the third is a power of ten multipuer as shown in figure. Hence a diode acts like a one – way value in circuit. Diodes are used to perform a variety of logic function in computer circuitry.

1.6 Electromotive Force and Circuits

For a conductor to have a steady current, it must be part of a path that forms a closed loop or complete circuit. Here's why, If you establish an electric field $\vec{E}_1$ inside an isolated conductor with resistivity ρ that is not part of a complete circuit, a current begins to flow with current density $\vec{J} = \frac{\vec{E}_1}{\rho}$. As a result a net positive charge quickly accumulates at one end of the conductor, and a net negative charge accurate the direction opposite to $\vec{E}_1$ causing the total electric field and hence the current to decrease. Within a very small fraction of a second enough charge builds up on the conductor ends that total electric field $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ inside the conductor. Then $\vec{J} = 0$ as well, and the current stops altogether. So there can be steady motion of charge in such an incomplete circuit.

As described in there is always a decrease in potential energy when charge part of the circuit in which the potential energy increase.

1.7 Electromotive Force

A charge travels "uphill." From lower to higher potential energy, even though the electrostatic force is trying to push ot from higher to lower potential energy. The direction of current in such a device is from lower to higher to higher potential, just the opposite of what happens in an ordinary conductor. The influence that vied emf and pronounced "ee – em – eff".

1.8 Internal Resistance

Real sources in a circuit don't behave in exactly the way we have described the potential difference across a real source in a circuit is not equal to the emf. The reason is that charge moving through the material of any real source encounters resistance. We call this internal resistance of the source, denoted by r, If this resistance behaves according to Ohm's Law r is constant and independent of the equal to Ir .Thus when a current is flowing through a source, the potential difference V_{ab} between the terminals of the source is:

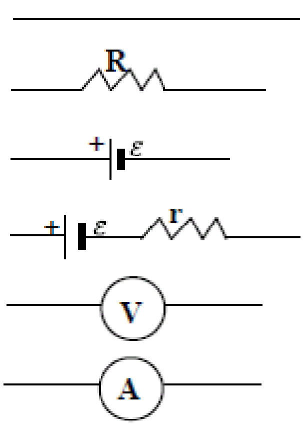
$$V_{ab} = \varepsilon - Ir \qquad \text{(terminal voltage, source with internal resistance)}$$

The potential V_{ab} called the terminal voltage is less than the emf ε because of the terms Ir representing the potential drop across the internal resistance r.

$$\varepsilon - Ir = IR$$

$$I = \frac{\varepsilon}{R + r} \qquad \text{(current source with internal resistance)}$$

1.9 Symbols for circuit Diagrams



Conductor with negligible resistance

Resistor

Source of emf

Source of emf with internal resistance *r*

Voltmeter

Ammeter

An important part of analyzing any electric circuit is drawing a schematic circuit diagram. We will use these symbols extensively in this chapter and the next. We usually assume that the wires that connect the various elements of the circuit have negligible resistance from last equation V = IR, the potential difference between the ends of such a wire is zero. A voltmeter, introduced in the table measures the potential difference between its terminals; an idealized voltmeter has infinitely large resistance and measures potential difference without having any current diverted through it. An ammeter measures the current passing through it; an idealized ammeter has zero resistance and has no potential difference between its terminals. Because meters act as part of the circuit in which they are connected, these properties are important to remember.

1.10 Energy and Power in Electric Circuits

Let's now look at some energy and power relations in electric circuits. The box in figure represents a circuit element with potential difference $V_a - V_b = V_{ab}$ between its terminals and current I passing through it in the direction from a toward b. This element might be a resistor, a battery, or something else.

The total work done on a charge q passing through the circuit element is equal to the product of q and the potential difference V_{ab}. When V_{ab} is positive, the electric force does a positive amount of work qV_{ab} on the charge as it "falls" from potential V_a to lower potential V_b. If the current is I, then in a time interval dt, an amount of charge dQ = I dt passes through. The work dW done on this charge is

$$dW = V_{ab}dQ = V_{ab}I\ dt$$

This work represents electrical energy transferred into this circuit element. The time rate of energy transfer is power, denoted by P.

$$\frac{dW}{dt} = P = V_{ab}I \quad \text{(rate of delivering electrical energy to a circuit element)},$$

If may happen that the potential at b is higher than that at a; then V_{ab} is negative, and there is a net transfer of energy out of the circuit element. The element is then acting as a source, delivering electrical energy into the circuit in which it is connected. This is the usual situation for a battery. Which converts chemical energy into electrical energy and delivers it to the external circuit.

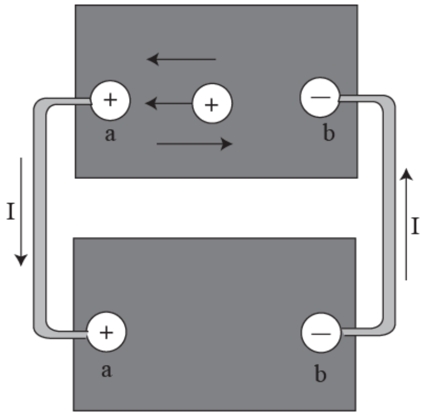
The unit of V_{ab} is one volt, or one joule per coulomb, and the unit of I is one ampere, or one coulomb per second. We can confirm that the SI unit of power is one watt:

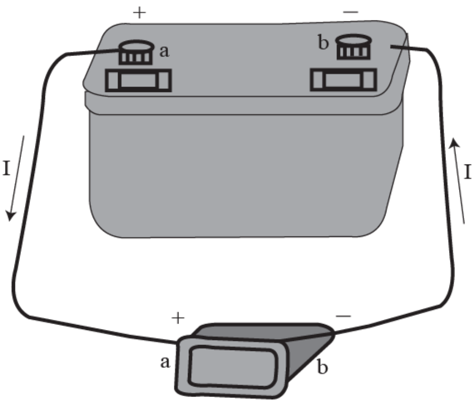
(1 J/C) (1 C/s) = 1 J/s = 1W.

Let's consider a few special cases.

1.11 Pure Resistance

If the circuit element in figure is a resistor, the potential difference is $V_{ab} = IR$. From the last equation the electrical power delivered to the resistor by the circuit is





$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R} \qquad \text{(power delivered to a resistor)}$$

In this case the potential at a is always higher than at b. Current enters the higher-potential terminal of the device, and the last equation represents the rate of transfer of electric potential energy into the circuit element.

1.12 Types of Electric Current

1. Steady state current or constant current

$$q = it$$

2. Transient or variable current

$$i = f(t)$$

$$i = \lim_{\Delta \to 0} \frac{\Delta q}{\Delta t} \quad \Rightarrow \quad \frac{dq}{dt}$$

3. Average Current

$$\boldsymbol{i} = \frac{\int_0^t \boldsymbol{i\,dt}}{\int_0^t \boldsymbol{dt}}$$

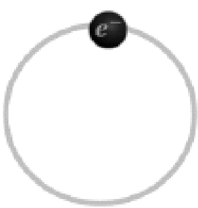
4. Convection current: Due mechanical transfer of charged particle

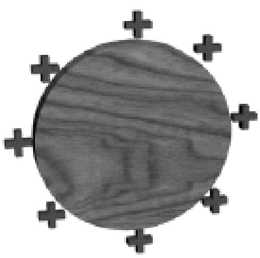
(a) Rotation point charge

$$i = \frac{q}{t}$$

$$T = \frac{2\pi}{\omega}$$

$$i = \frac{q\omega}{2\pi}$$



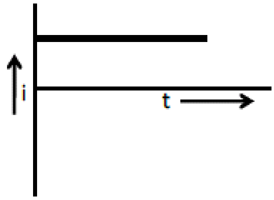


(b) Rotating non-conducting ring

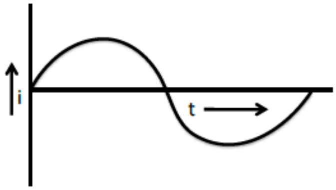
$$i = \frac{2\pi R\lambda\omega}{2\pi}$$

$$i = R\lambda\omega$$

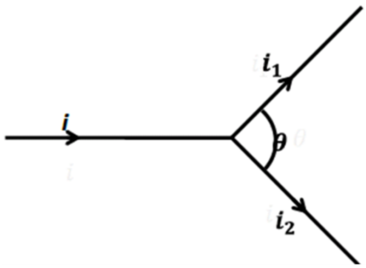
5. Direct Current



6. Alternating Current



1.13 Electric Current Vector or Scalar

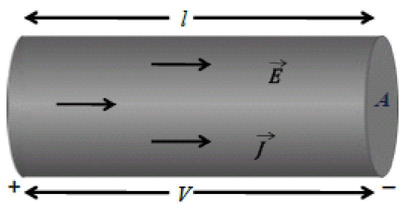


1.14 Ohm's Law

$\vec{E} \propto \vec{J}$

$\vec{E} = \rho\vec{J}$

$\frac{V}{l} = \frac{El}{JA} = \rho\frac{l}{A}$



$\rho\frac{l}{A}$ is constant for ohmic materials. This is called the resistance R.

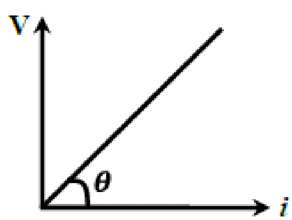
$R = \frac{V}{i}$

$R = \rho\frac{l}{A}$

$V = iR$

$R = \frac{V}{i} = \tan\theta$

$G = \frac{1}{R} = \frac{l}{V}$ Conductance



G is ohm^{-1}

Resistivity

$$\rho = \frac{E}{l}$$

SI units of resistivity are Ω-m (ohm-meter). A perfect conductor would have zero resistivity, and a perfect insulator would have an infinite resistivity.

The reciprocal of resistivity is conductivity (σ)kk

$$\sigma = \frac{1}{\rho}$$

Its units are $(\Omega - \text{m})^{-1}$.

Illustration

Illustration

If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?

Solution

As fro a given wire,

$$R = \rho\frac{L}{S} \text{ with } L \times S = \text{volume} = V = \text{constant}$$

So that $R = \rho\left[\frac{L^2}{V}\right]$ $\left[\text{as } S = \frac{V}{L}\right]$

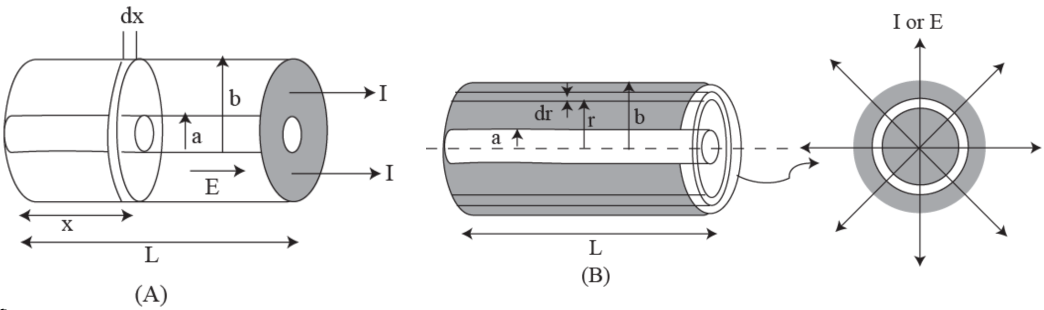
And hence fraction change in resistance with change in length

$\left[\text{provided } \left(\frac{\Delta L}{L}\right) \text{is small say} < 10\%\right]$ when its volume remains constant:

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2(0.1\%) = 0.2\%$$

Illustration

A cylindrical tube of length L has inner radius a while outer b as show in figure (A). What is the resistance of the tube between (a) its ends (b) its inner and outer surface? [The resistivity of its material is ρ.]



Solution

(a) In case of electric conduction, field at a point inside a conductor is given by,

$$J = \sigma E, \quad i.e., \quad E = \rho J \qquad \left[\text{as } \sigma = \left(\frac{1}{\rho}\right)\right] \qquad \ldots(1)$$

As here the current add field are along the axis of the tube, consider the tube to be made up large number of co – axial annular discs and considering a disc of thickness dx at distance x from one end as shown in figure (A) we have,

$$E = \frac{dV}{dx} \quad and \quad J = \frac{I}{\pi(b^2 - a^2)} \qquad [\text{as } S = \pi(b^1 - a^2)]$$

So equation (1) reduces to:

$$-\frac{dv}{dx} = \rho \frac{1}{\pi(b^2 - a^2)}, \quad i.e., \quad -\int_{V_1}^{V_2} dV = \int_0^L \frac{\rho I dx}{\pi(b^2 - a^2)}$$

or $\quad V = (V_1 - V_2) = \frac{\rho L I}{\pi(b^2 - a^2)}$ So $\quad R = \frac{V}{I} = \frac{\rho L}{\pi(b^2 - a^2)}$

(b) As here the field is radial, consider the tube to the made up of large number of concentric cylindrical shells and considering a shell of radius r and thickness dr as shown in figure (B), we have,

$$E = -\frac{dv}{dr} \text{ and } J = \frac{I}{2\pi r L} \qquad [\text{as here } S = 2\pi r L]$$

So equation (1) fro this case becomes,

$$-\frac{dV}{dr} = \frac{\rho I}{2\pi L}, \quad i.e., \quad -\int_{V_1}^{V_2} dV = \int_a^b \frac{\rho I dr}{2\pi r L}$$

or, $\quad V = (V_1 - V_2) = \frac{\rho I}{2\pi L} \log_e \left[\frac{b}{a}\right]$ So, $\quad R = \frac{V}{I} = \frac{\rho}{2\pi L} \log_e \left[\frac{b}{a}\right]$

Illustration

A potential difference 30 V is applied between the ends of a conductor of length 100 m and resistance 5Ω.The conductor has a uniform cross – section. Find the total linear momentum of free electrons?

Solution

Potential difference applied across the conductor results in a current:

$$I = \frac{V}{R} = \frac{30}{0.5} = 60\ A$$

Free electrons drift towards higher potential with an average velocity $\overrightarrow{V_d}$. Linear momentum of each electron will be $m\overrightarrow{V_d}$. Let n be the free electron density so that total number of free electrons = *nAl*, where A is the area of cross – section and l the length of conductor.

Since the drift momentum of each electron is in the same direction, magnitude of linear momentum of all electrons.

$$P = (vAl)(mv_d)$$

But $\quad I = nAev_d \quad$ so that, $\quad nav_d = \frac{1}{e}$

Hence $\quad P = \frac{Ilm}{e}$

$$I = 60A \quad I = 100\ m \quad m = 9.1 \times 10^{-31}\ kg \quad e = 1.6 \times 10^{-19}\ C$$

$$P = \frac{60 \times 100 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} = 3.4 \times 10^{-8}\ kg/sec$$

Question Practice Online