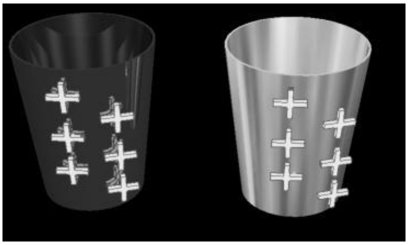
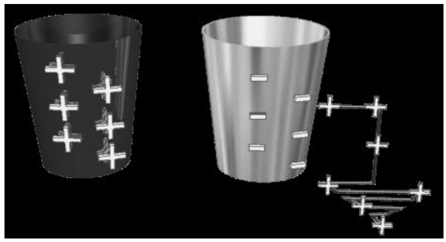
Chapter 2 CAPACITOR

Day-1

1. Capacitor

Any two conductor having opposite charge place close to each other and if they maintain a constant potential difference across them is known as capacitor.

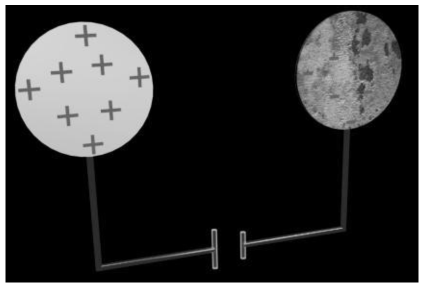




1.1 Objectives of Capacitors

(i) To collect charge or potential energy.

(ii) To maintain potential difference constant.



$$q \propto V$$

$$\Rightarrow \quad \frac{q}{V} = \text{constant}$$

$$\Rightarrow \quad \frac{q}{V} = C$$

Capacitor defined as:

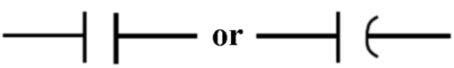
(i) Ability to store the charge and potential energy.

(ii) Ability to maintain potential difference.

1.2 Units Capacitance

Farad (f), Pico Farad (pf) etc.

Symbol



1.3 Capacitors and Capacitance

Any two conductors separated by an insulator form a capacitor. In most practical applications, each conductor initially has zero net charge and electrons are transferred from one conductor to the other; this is called charging the capacitor. Then the two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero. We will assume throughout this chapter that this is the case. When we say that a capacitor has charge Q, or that a charge Q is stored on the capacitor, we mean that the conductor at higher potential has charge +Q and the conductor at lower potential has charge –Q. Keep this in mind in the following discussion and examples.

In circuit diagrams a capacitor is represented by either of these symbols:

In either symbol the vertical lines represent the conductors and the horizontal lines represent wires connected to either conductor. One way to charge a capacitor is to connect these two wires to opposite terminals of a battery.

The electric field at any point in the region between the conductors is proportional to the magnitude Q of charge on each conductor. It follows that the potential difference V_{ab} between the conductors is also proportional to Q. If we double the magnitude of charge on each conductor, the charge density at each point doubles, the electric field at each point doubles, and the potential difference between conductors doubles; however, the ratio of charge to potential difference does not change. This ratio is called the capacitance C of the capacitor:

$$C = \frac{q}{V_{ab}} \qquad \text{(definition of capacitance)}$$

The SI unit of capacitance is called one farad (1 F), in honor of the nineteenth century English physicist Michael Faraday. From the last equation, one farad is equal to one coulomb per volt (1 C/V): 1 F = 1 farad = 1 coulomb/volt)

The simplest form of capacitor consists of two parallel conducting places, each with area A, separated by a distance d that is small in comparison with their dimensions. When the plates are charged, the electric field is almost completely localized in the region between the plates. We call this arrangement a parallel-plate capacitor.

We worked out the electric-field magnitude E for this arrangement in Example using the principle of superposition of electric fields. So the field magnitude E can be expressed as

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The field is uniform, and the distance between the plates is d, so the potential difference between the two plates is

$$V_{ab} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

From this we see that the capacitance C of a parallel-plate capacitor in vacuum is

$$C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d} \quad \text{(capacitance of a parallel} - \text{plate capacitor in vacuum)}$$

1.4 Actual Capacitor

Applications of capacitor

Radio receivers, Television receivers, Air bag sensors for car electronic flash units for photography etc.

1.5 Types of Capacitors

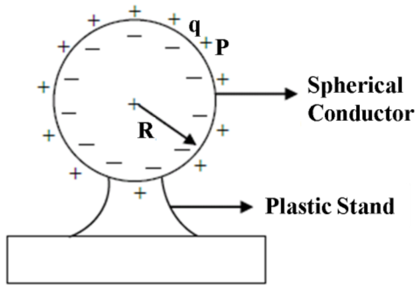
(i) Isolated Spherical Capacitor

Potential at point P

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

$$C = \frac{q}{V} = \frac{q4\pi\epsilon_o}{q}$$

$$C = 4\pi\epsilon_o R$$



(ii) Earthed Spherical Capacitor

Potential at $P_1 \Rightarrow V_{P_1} = 0$

Potential at $P_2 \Rightarrow V_{P_2} \Rightarrow$

$$V_{P_2} = \frac{1}{4\pi\epsilon_o}\left(\frac{q}{a} - \frac{q}{b}\right)$$

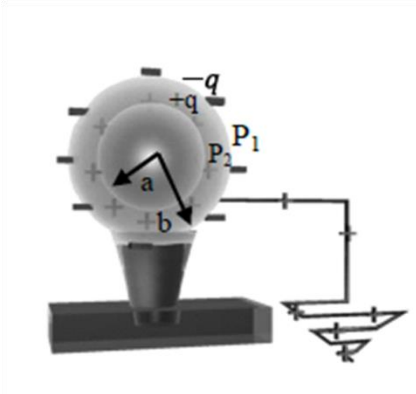
$$\Delta V = V_{P_2} - V_{P_1}$$

$$= \frac{q}{4\pi\epsilon_o}\left(\frac{b-a}{ab}\right)$$

$$C = \frac{q}{\Delta V}$$

$$C = \frac{q\,ab}{q(b-a)}4\pi\epsilon_o$$

$$C = 4\pi\epsilon_o\frac{ab}{b-a}$$



(iii) Earthed Concentric Spherical Capacitor

Potential $P_1 \Rightarrow V_P = 0$

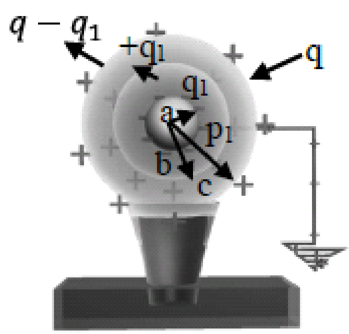
$$\frac{1}{4\pi\epsilon_o}\left[-\frac{q_1}{a} + \frac{q_1}{b} + \frac{q-q_1}{c}\right] = 0$$

$$q_1 = q\left(\frac{ab}{bc+ab-ac}\right)$$

$$\Delta V = \frac{1}{4\pi\epsilon_o}\left[\frac{q_1}{a} - \frac{q_1}{b}\right]$$

$$\Delta V = \frac{q_1}{4\pi\epsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Delta V = \frac{q_1}{4\pi\epsilon_o}\left(\frac{b-a}{ab}\right)$$



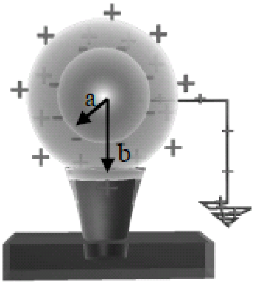
$$\Delta V = \frac{q}{4\pi\epsilon_o}\left(\frac{b-a}{bc+ab-ac}\right)$$

$$C = \frac{q}{\Delta V} = 4\pi\epsilon_o\ \left(\frac{bc+ab-ac}{b-a}\right)$$

$$= 4\pi\epsilon_o\left(\frac{ab}{b-a} + \frac{C(b-a)}{b-a}\right)$$

$$C = 4\pi\epsilon_o\left(\frac{ab}{b-a} + C\right)$$

(vi) Spherical Earthed Capacitor



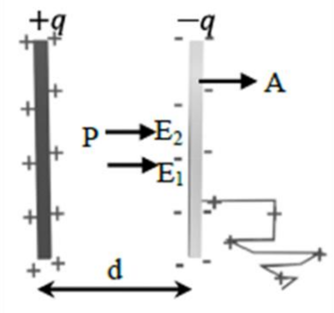
In pervious case

Put b = c

$$C = 4\pi\epsilon_o\left(\frac{ab}{b-a} + b\right)$$

$$C = 4\pi\epsilon_o\left(\frac{b^2}{b-a}\right)$$

(v) Parallel Plate Capacitor



Net electric field at point P

$$= E_1 + E_2$$

$$= \frac{q}{2\epsilon_o A} + \frac{q}{2\epsilon_o A}$$

$$E = \frac{q}{\epsilon_o A}$$

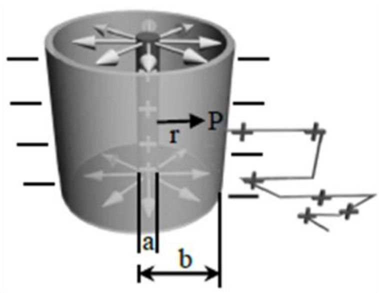
Now $E = \frac{V}{d} = \frac{q}{\epsilon_o A}$

$\Rightarrow \quad V = \frac{qd}{\epsilon_o A}$

Now $V = \frac{q}{C} \Rightarrow \frac{q}{C} = \frac{qd}{\epsilon_o A}$

$\Rightarrow \quad C = \frac{\epsilon_o A}{d}$

(vii) Cylindrical capacitor



Electric field at point P

$$E_P = \frac{\lambda}{2\pi\epsilon_o r}$$

Now $E_P = -\frac{dv}{dr}$

$$-\frac{dv}{dr} = \frac{\lambda}{2\pi\epsilon_o r}$$

$$\int_0^V dV = -\frac{\lambda}{2\pi\epsilon_o}\int_a^b \frac{dr}{r}$$

$$V = -\frac{\lambda}{2\pi\epsilon_o}\ \text{In}\ \frac{a}{b}$$

$$V = \frac{\lambda}{2\pi\epsilon_o}\ \text{In}\ \frac{b}{a}$$

Now $\lambda = \frac{a}{l}$

$$V = \frac{\frac{q}{l}}{2\pi\epsilon_o}\ \text{In}\frac{b}{a}$$

$C = \frac{q}{V}$ ⇒ Capacitance per unit length

$$C' = \frac{C}{l} = \frac{\frac{q}{l}}{V}$$

$$C' = \frac{\frac{q}{l}2\pi\epsilon_o}{\frac{q}{l}\text{In}\frac{b}{a}}$$

$$C' = \frac{2\pi\epsilon_o}{\text{In}\frac{b}{a}}$$

1.6 Dielectrics

Most capacitors have a non conducting material or dielectric, between their conducting plates.

1.7 Advantages of Dielectrics

(i) It maintain two large metal conducting sheets at a very small separation without actual contact.

(ii) It allows a capacitor to sustain a higher potential difference V and so store greater amounts of charge and energy.

(iii) Increase the capacitance K times

$$C_m = K\,C_{\text{air}}$$

↓

In any medium

Here K is dielectric constant

Note:- In presence of any dielectric between plates.

(i) Capacitance increase by K times.

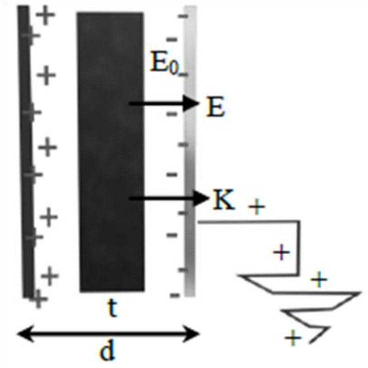
(ii) Electric field and Potential difference decrease by K times.

(iii) Amount of charge remains constant value of K (dielectric constant)

(1) Vacuum	K = 1.
(2) Air (1atm)	K = 1.00059 ⇒ K = 1
(3) Teflon	K = 2.1
(4) Mica	K = 3 to 6
(5) Glass	K = 5 to 10
(6) Glycerin	K = 42.5

(7) Water K = 80.4

1.8 Capacitor Partially Filled With Dielectric



Charge on dielectric due to induction

$$q_1 = q\left(1 - \frac{1}{K}\right)$$

Potential difference between plates of the capacitor

$$V = Et + E_o(d - t)$$

$$V = \frac{E_o}{K}t + E_o(d - t)$$

$$= E_o\left((d - t) + \frac{t}{K}\right)$$

$$V = E_o\left[(d - t) + \frac{t}{K}\right]$$

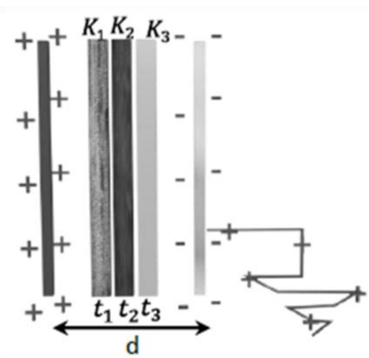
$$V = \frac{q}{\epsilon_0 A}\left[(d - t) + \frac{t}{K}\right]$$

$$C = \frac{q}{V} = \frac{q\epsilon_0 A}{q\left[(d-t)+\frac{t}{K}\right]}$$

$$C = \frac{\epsilon_0 A}{\left[(d-t)+\frac{t}{K}\right]}$$

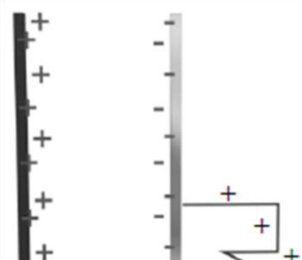
(i) If n dielectrics

$$C = \frac{\epsilon_0 A}{\left[\left(d - (t_1 + t_2 + \dots + t_n)\right) + \left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \dots + \frac{t_n}{k_n}\right)\right]}$$



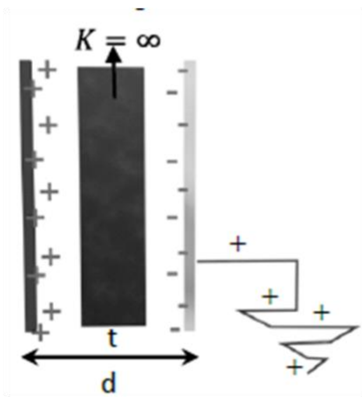
(ii) If completely filled by dielectric

$$C = \frac{K\epsilon_0 A}{d}$$



(iii) If conducting slab ($K = \infty$) placed between plates

$$C = \frac{\epsilon_0 A}{d - t}$$



1.9 Electric-Field Energy

We can charge a capacitor by moving electrons directly from one plate to another. This requires doing work against the electric field between the plates. Thus we can think of the energy as being stored in at the field in the region between the plates. To develop this relation, let's find the energy per unit volume in the space between the plates of a parallel-plate capacitor with place area A and separation d. We call this the energy density, denoted by u. From the last equation the total stored potential energy is $\frac{1}{2}CV^2$ and the volume between the plates is just Ad; hence the energy density is

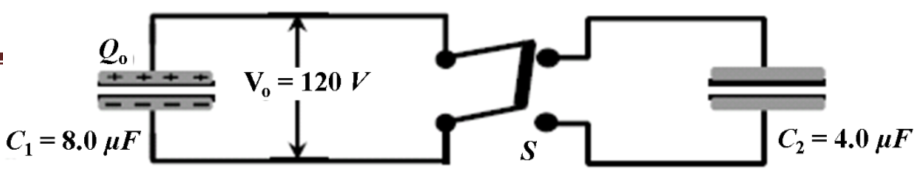
$$u = \text{Energy density} = \frac{\frac{1}{2}CV^2}{Ad}$$

From the last equations the capacitance C is given by C = ε_o A/d. The potential difference V is related to the electric field magnitude E by V = Ed. If we use these expressions in the last equation, the geometric factors A and d cancel, and we find

$$u = \frac{1}{2}\varepsilon_o E^2 \quad \text{(electric enrgy density in a vacuum)}$$

Illustration

In figure we charge a capacitor of capacitance $C_1 = 8\ \mu F$ by connection it to a source of potential difference $V_o = 120\ V$. The switch S is initially open. Once C_1 is charged, the source of potential difference is disconnected. (a) What is the charge Q_o on C_1 if switch S is left open? (b) What is the



energy stored in C_1 if switch S is left open? (c) The capacitor of capacitance C_2 = 4.0 μF is initially uncharged. After we close switch S, what is the potential difference across each capacitor, and what is the charge on each capacitor? (d) What is the total energy of the system after we close switch S?

Solution

(a) The charge Q_o on C_1 is

$$Q_o = C_1 V_o = (8.0\ \mu F)(120\ V) = 960\ \mu C$$

(b) The energy initially stored in the capacitor is

$$U_{initial} = \frac{1}{2}\ Q_o V_o = \frac{1}{2}\ (960 \times 10^{-6}\ C)(120\ V)$$

(c) When the switch is closed, the positive charge Q_o becomes distributed over the upper plates of both capacitors and the negative charge $-Q_o$ is distributed over the lower plates of both capacitors. Let Q_1 and Q_2 be the magnitudes of the final charges on the two capacitors. From conservation of charge,

$$Q_1 + Q_2 = Q_o$$

$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$V = \frac{Q_o}{C_1 + C_2} = \frac{960\ \mu C}{8.0 \mu F + 4.0\ \mu F} = 80\ V$$

$$U_{final} = \frac{1}{2} Q_1 V + \frac{1}{2} Q\ \ V += \frac{1}{2} Q_o V$$

$$= \frac{1}{2}(960 \times 10^{-6})(80\ V) = 0.038\ \mathrm{J}$$

1.10 Dielectrics

Most capacitors have a no conducting material, or dielectric, between their conducting plates. A common type of capacitor uses long strips of metal foil for the plates, separated by strips of plastic sheet such as Mylar. A sandwich of these materials is rolled up, forming a unit that can provide a capacitance of several microfarads in a compact package.

Placing a solid dielectric between the plates of a capacitor serves three functions. First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.

Second, using a dielectric increases the maximum possible potential difference between the capacitor plates.

Third, the capacitance of a capacitor of given dimensions is greater when there is a dielectric material between the plates than when there is a vacuum. We can demonstrate this effect with the aid of a sensitive electrometer, a device that measures the potential difference between two conductors without letting any appreciable charge flow from one to the other.

The original c capacitance C_o is given by $C_o = Q/V_o$, and the capacitance C with the dielectric present is C = Q/V. The charge Q is the same in both cases, and V is less than V_o, so we conclude that the capacitance C with the dielectric present is greater than C_o. When the space between plates is completely filled by the dielectric, the ratio of C to C_o is called the dielectric constant of the material, K:

$$K = \frac{C}{C_o} \qquad \text{(definition of dielectric , constant)}$$

With the charge is constant, $Q = C_oV_o = CV$ and $\frac{C}{C_o} = \frac{V_o}{V}$. In this case, equation can be rewritten as

$$V = \frac{V_o}{K} \qquad \text{(when Q is costant)}$$

With the dielectric present, the potential difference for a give charge Q is reduced by a factor K.

1.11 Values of Dielectric Constant K at 20º C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air(1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5-10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3-6	Water	80.4
Mylar	3.1	Strontium titanate	310

1.12 Induced Charge and Polarization

When a dielectric material is inserted between the plates while the charge is kept constant, the potential difference between the plates decreases by a factor K. Therefore the electric field between the plates must decrease by the same factor.

If Eo is the vacuum value and E is the value with the dielectric, then

$$E = \frac{E_o}{K} \qquad \text{(when Q is constant)}$$

Since the electric-field magnitude is smaller when the dielectric is present, the surface charge density (which causes the field) must be smaller as well. The surface charge on the conducting plate does not change, but an induced charge of the opposite sign appears on each surface of dielectric. The dielectric was originally electrically neutral, and is still neutral; the induced surface charges arise as a result of redistribution of positive and negative charge within the dielectric material, a phenomenon called polarization.

We can derive a relation between this induced surface charge and the charge and the charge on the plates. Let's denote the magnitude of the charge per unit area induced on the surfaces of the dielectric by σ_1. The magnitude of the surface charge density on the capacitor plates is σ, as usual. Then the net surface charge on each side of the capacitor has magnitude ($\sigma - \sigma_i$) as shown in figure. Without and with the dielectric, respectively, we have

$$E_o = \frac{\sigma}{\epsilon_o} \qquad E = \frac{\sigma - \sigma_i}{\epsilon_o}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \qquad \text{(induced surface charge density)}$$

This equation shows that when K is very large, σ_i is nearly as large, σ as. In this case, σ_i nearly cancels σ, and the field and potential difference are much smaller than their values in vacuum.

The product $K\varepsilon_o$ is called the permittivity of the dielectric, denoted by ε:

$$\varepsilon = K\varepsilon_o \qquad \text{(definition of premittivity)}$$

$$E = \frac{\sigma}{\varepsilon}$$

The capacitance when the dielectric is present is given by

$$C = KC_o = K\varepsilon_o\frac{A}{d} = \varepsilon\frac{A}{d} \qquad \text{(parallel − plate capacitor, dieelectric between plates)}$$

We can repeat the derivation of above equations for the energy density u in an electric field for the case in which a dielectric is present. The result is

$$u = \frac{1}{2}K\varepsilon_o E^2 = \frac{1}{2}\varepsilon E^2 \qquad \text{(electric energy density in a dielectric)}$$

Illustration

Suppose the parallel plates each have an area of 200 cm^2 (2.00×10^{-1} m^2) are the 1.00cm (1.00×10^{-2} m^2) aprt. The capacitor is connected to a power supply and charged to a potential difference V_o = 300 *kV*. It is then disconnected from the power supply, and completely filling the space between them. We find that the potential difference decreases to 1000V while the charge on each capacitor plate remains constant. Compute a); the original capacitance C_o. b); the magnitude of charge Q on each plate c); the after the dielectric is inserted d); the dielectric constant K of the dielectric e); the permittivity ∈ of the dielectric f); the magnitude of the induced charg Q_i on each face of the dielectric g); the original electric field E_o between the plates and h); the electric field E after the dielectric is inserted?

Solution

a) With vacuum between the plates, we use with K = 1

$$C_o = \epsilon_o\frac{A}{d} = (8.85 \times 10^{-12}\ F/M)\ \frac{2.00 \times 10^{-1}\ m^2}{1.00 \times 1-^{-2}\ m}$$

$$= 1.77 \times 10^{-10}F = 177\ pF$$

b) Using the dielectric of capacitance,

$$Q = C_oV_o = (1.77 \times 10^{-10}F)(3.00 \times 10^3V)$$

$$= 5.31 \times 10^{-7}C = 0.531\ \mu C$$

c) When the electric is inserted, the charge remains the same but the potential decreases to V =

1000V. Hence the new capacitance is

$$\boldsymbol{C} = \frac{\boldsymbol{Q}}{V} = \frac{5.31 \times 10^{-7}\ C}{1.00 \times 10^{-10}\ V} = 5.31 \times 10^{-10} = 531\ F$$

d) The dielectric constant is

$$K = \frac{C}{C_o} = \frac{5.31 \times 10^{-7}\ F}{1.77 \times 10^{-10}\ F} = \frac{531\ pF}{177\ pF} = 3.00$$

$$K = \frac{V_o}{V} = \frac{3000\ V}{1000\ V} = 3.00$$

e) Using K from park (d) in, the permittivity is

$$\in = K_{\in_o} = (3.00)\left(8.85 \times 10^{-12}\frac{c^2}{N}.m^2\right)$$

$$= 2.66 \times 10^{-11}\ C^2/N.m^2$$

f) $$Q_i = Q\left(1 - \frac{1}{K}\right) = (5.31 \times 10^{-7}\ C)\ \left(1 - \frac{1}{3.00}\right)$$

$$= 3.54 \times 10^{-7} C$$

g) $$E_o = \frac{V_o}{d} = \frac{3000\ V}{1.00 \times 10^{-2}} = 3.00 \times 10^5\ V/m$$

h) With the new potential difference after the dielectric is inserted,

$$E = \frac{V}{d} = \frac{1000\ V}{1.00 \times 10^{-2}\ m} = 1.00 \times 10^5\ V/m$$

$$E = \frac{\sigma}{\in} = \frac{Q}{\in A} = \frac{5.31 \times 10^{-7} C}{(2.66 \times 10^{11} C^2/N.m)\ (2.00 \times 10^{-1}\ m^2)}$$

$$= 1.00 \times 10^5\ V/m$$

$$E = \frac{\sigma - \sigma_i}{\in_o} = \frac{Q - Q_i}{\in_o A}$$

$$= \frac{(5.31 - 3.54) \times 10^{-7}\ C}{(8.85{\times}10^{-12}\ C^2 N/m^2)\ (2.00\ {\times}10^{-1}\ m^2)}$$

$$= 1.00 \times 10^5\ V/m$$

$$E = \frac{E_o}{K} = \frac{3.00 \times 10^5\ V/m}{3.00} = 1.00 \times 10^5\ V/m$$

1.13 Dielectric Breakdown

We mentioned earlier that when any dielectric material is subjected to a sufficiently strong electric field, dielectric breakdown takes place and the dielectric becomes a conductor. Thus occurs when the electric field is so strong that electric are ripped loose from their molecules and crash into other molecules, liberating even more electrons. This avalanche of moving charge, forming a spark or arc discharge, often starts quite suddenly. Because of dielectric breakdown, capacitors always have maximum voltage ratings. When a capacitor is subjected to excessive voltage, an arc may from through a layer of dielectric, corning or melting a hole in it.

Dielectric Constant and Dielectric Strength of some insulating Materials

Material	Dielectric Constant. K	Dielectric Strength, $E_{\max}(V/m)$
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex Glass	4.7	1×10^7

Illustration

Illustration

The plates of a parallel-plate capacitor in vacuum are 5.00 mm apart and 2.00 m^2 in area. A potential difference of 10,000 V (10.0 kV) is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field in the space between them.

Solution

(a) From

$$C = \varepsilon_o \frac{A}{d} = \frac{\left(8.85 \times 10^{-17} \frac{F}{m}\right)(2.00\ m^2)}{5.00 \times 10^{-3}}$$

$$= 3.54 \times 10^{-9}\ F = 0.00354\ \mu F$$

(b) The charge on the capacitor is

$$Q = CV_{ab} = \left(3.54 \times 10^{-9} \frac{C}{V}\right)(1.00 \times 10^4\ V)$$

$$= 3.54 \times 10^{-5} C = 35.4\ \mu C$$

The plate at higher potential has charge +35,4 μC and the other plate has charge –35.4 μC.

(c) The electric-field magnitude is

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_o A} = \frac{3.54 \times 10^{-5}}{\left(8.85 \times 10^{-12} \frac{C^2}{N}.m^2\right)(2.00\ m^2)}$$

$$= 2.00 \times 10^6\ N/C$$

Question Practice Online