Chapter 1

ELECTROSTATE

Day – 1

Electric Charge

Electromagnetic interactions involve particles that have a property called electric charge, an attribute that is as fundamental as mass. Just as objects with mass are accelerated by gravitational forces, so electrically charged objects are accelerated by electric forces. The annoying electric spark you feel when you scuff your shoes across a carpet and then and then reach for a metal doorknob is due to charged particles leaping between your finger and the doorknob. Even the forces that hold atoms together to form solid matter, and that keep the atoms of solid objects from passing through each other, are fundamentally due to electric interactions between the charged particles within atoms.

1. Electric Charge

The ancient Greeks discovered as early as 600 B.C. that they rubbed amber with wool, the amber could then attract other objects. Today we say that the amber has acquired a net electric charge, or has become charged. The word ―electric‖ is derived from the Greek word elektron, meaning amber.

Plastic rods and fur (real or fake) are particularly good for demonstrating electrostatics, the interactions between electric charges that are at rest. Figure shows two plastic rods and a pies of fur. After we charge each rod by rubbing it with the piece of fur, we find that the rods repel each other.

When we rub glass rods with silk, the glass rods also become charged and repel each other. But a charged plastic rod attracts a charged glass rod. Furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other.

These experiments and many others like them have shown that there are exactly two kinds of electric charge : the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706-1790) suggested calling these two kinds of charge negative and positive, respectively and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge. Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.

1.1 Electric Charge and the Structure of Matter

The structure of atoms can be described in terms of three particles: the negatively charged electron, the positively charged proton, and the uncharged neutron. The proton and neutron are combinations of other entities called quarks, which have charges $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ of times the electron charge.

The protons and neutrons in an atom make up a small, very dense core called the nucleus, with dimensions of the order of 10^{-15} m. Surrounding the nucleus are the electrons, extending out to distances of the order of 10^{-10} m from the nucleus. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus.

The masses of the individual particles, to the precision that they are presently known, are

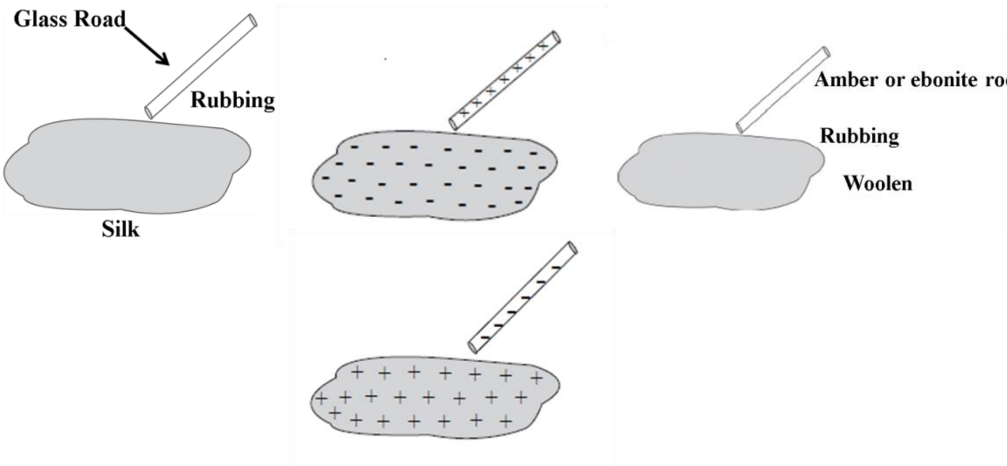
Mass of electron $= m_e = 9.10938188\ (72) \times 10^{-31}$Kg

Mass of proton = m_p = 1.67262158 (13) × 10^{-27} Kg

Mass of neutron = m_n = 1.67492716 (13) × 10^{-27} Kg

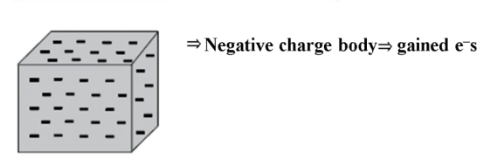
1.2 The study of electrical effect of charge at rest.

1.3 Electric Charge

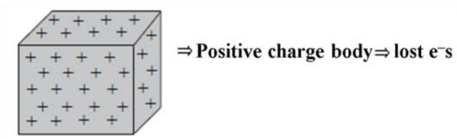


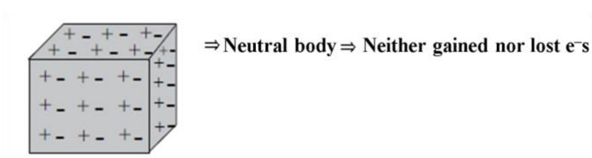
1.4 Charged Body

Excess of electrons



Deficiency of electron





1.5 Amount of Charge

$$Q = \pm ne$$

Here n → No. of electrons gained or lost

$e^{-} \rightarrow 1.6 \times 10^{-19}$ Coulomb

+ sign for e⁻ emitter and – sign for e⁻ acceptor

1.6 Units of charge

SI Unit → ampere × Sec

M. K. S Unit → Coulomb

Smallest Unit → Stiatic Coulomb (esu) of frakline

Largest Unit → Faraday

1.7 Conversion

$1 \text{ Coul.} = 3 \times 10^{9} \text{esu}$

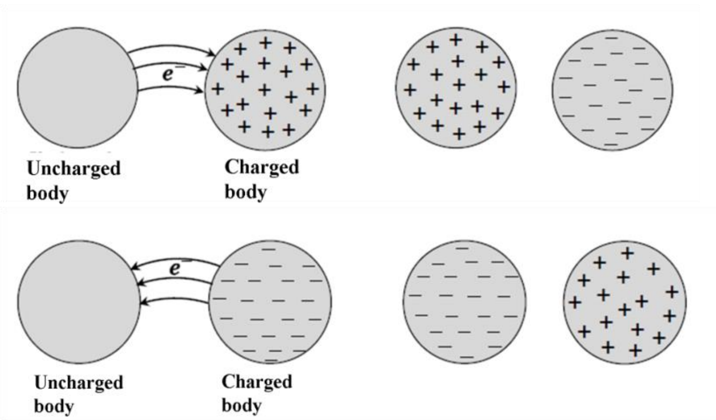
$1 \text{ Coul.} = 1\text{ampere} \times \text{sec}$

$1 \text{ Faraday} = 96500 \text{ coul.}$

1.8 Ways to Charging a body

(i) Conduction

Transfer of electron due to contact of two bodies



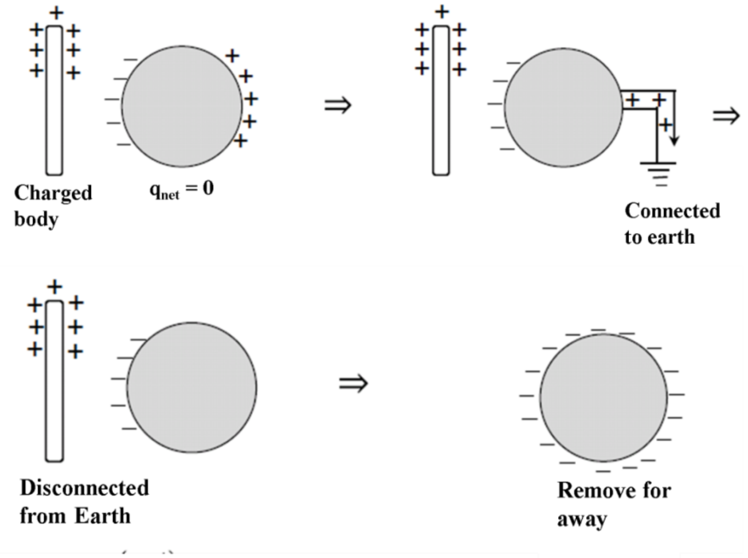
Note:-will flow between two bodies until potential of both becomes equal mass of both bodies will change also in this process of charging.

(ii) Friction

Transfer of electron due to rubbing of two bodies.

(iii) Electrical induction

Charging of bodies without transfer of electron only configuration of charge will change when placed very close.



$$q_i = \pm q\left(1 - \frac{1}{k}\right)$$

Here $q_i \rightarrow$ Amount of induced charge

q $\rightarrow$ Charge on original body

k $\rightarrow$ dielectric constant of uncharged body

If uncharged body is metals then k = ∞ ⇒ $q_i = \pm q\left(1 - \frac{1}{\infty}\right) = \pm q$

If uncharged body is perfect insulator $k = 0,\ \ q_i = \pm q\left(1 - \frac{1}{\infty}\right) = \pm q$

1.9 Properties of Charges

(i) Charge is quantized

By Millikan‘s oil drop experiment we must say the smallest charge that can exist in nature is the charge of an electron which is equal to 1.6×10^{-19} Coul.

$$q = \pm\, ne$$

Here n is an integer.

Note: But now it has been discovered that elementary particles such as proton or neutron are composed of quarks having fractional charge such as $\pm\frac{e}{3}, \pm\frac{5}{3}e,$ etc.

But these quark particles do not exist in free state.

(ii) Accelerated charge

Electromagnetic theory has established that charged particle at rest produce only electric field.

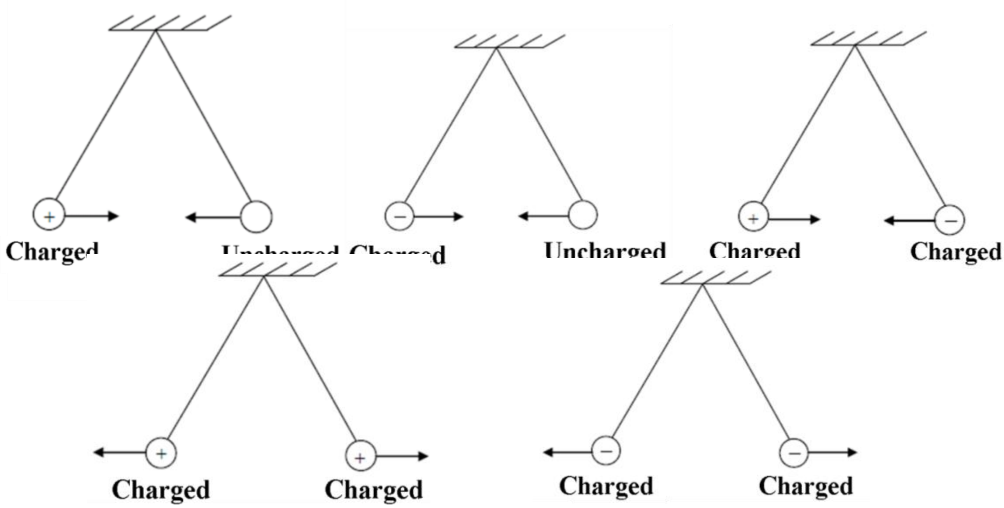
$\ominus \rightarrow$ Product E. F

At rest

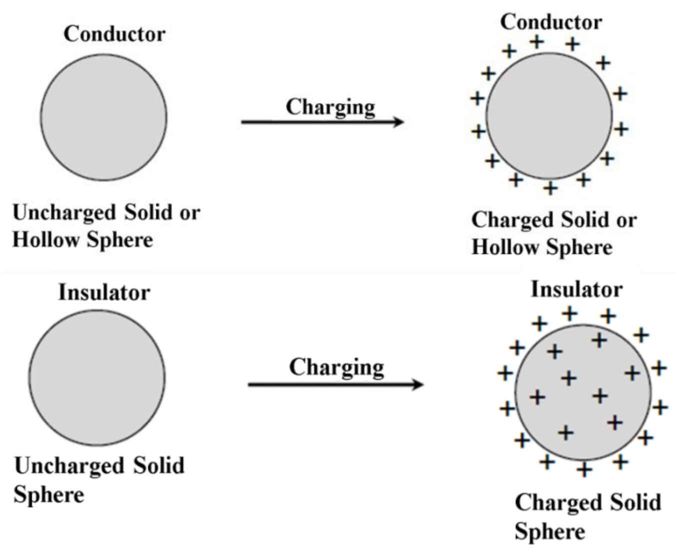
$\ominus \rightarrow V = \text{Constant} \rightarrow$ Produce E. F and M. F

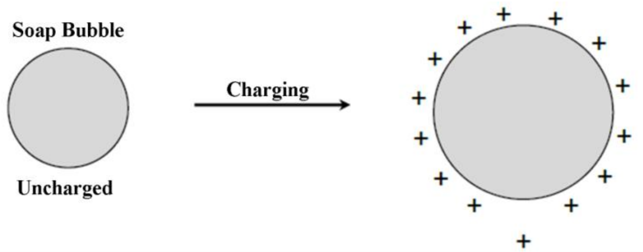
$\ominus \rightarrow V = \text{Change} \rightarrow$ Produce E. F and M. F. also radiate energy

(iii) Attraction or Repulsion



(iv) Charge resides on the outer surface of a conductor

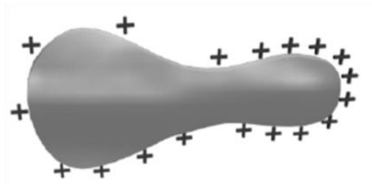




(v) Distribution of charge in conductors

Charge will be maximum at sharp corners because $\sigma \propto \frac{1}{R}$.

Also charge leaks from sharp points.



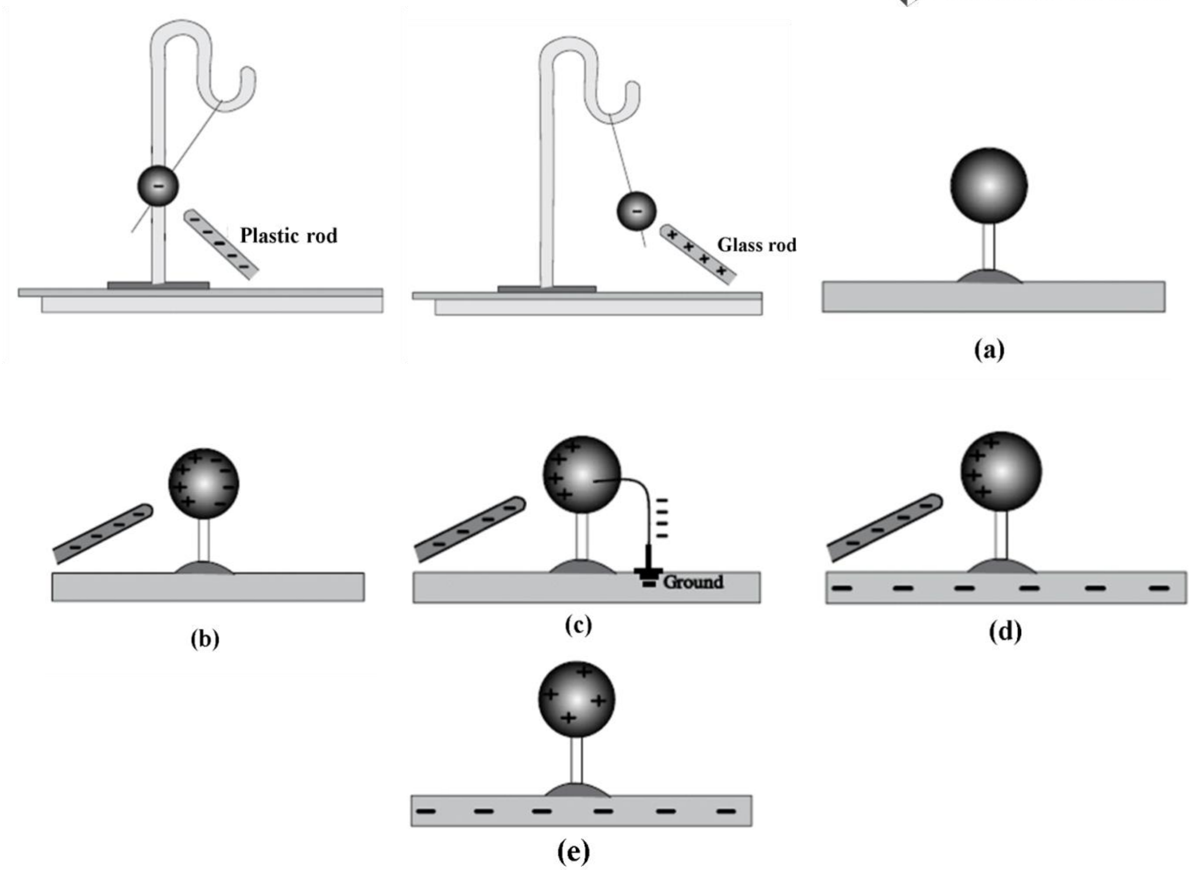
The negative charges of the electron has exactly the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge is exactly zero. The number of protons or electrons in a neutral atom of an element is called the atomic number of the element. If one or more electrons are removed, the remaining positively charged structure is called a positive ion. A negative ion is an atom that has gained one or more electrons. This gaining or losing of electrons is called ionization.

We can create an excess positive charge by either adding positive charge or removing negative charge. In most cases, negatively charged electrons are added or removed, and a "positively charged body" is one that has lost some of its normal complement of electrons.

Implicit in the foregoing discussion are two very important principles. First is the principle of conservation of charge: The algebraic sum of all the electric charges in any closed system is constant. The second important principle is that the magnitude of charge of the electron or proton is a natural unit of charge.

1.10 Induced Charges

We can charge a metal ball by touching it with an electrically charged plastic rod, as in figure. In this process, some of the excess electrons on the rod are transferred from it to the ball, leaving the rod with a smaller negative charge. There is a different technique in which the plastic rod can give another body a charge of opposite sign without losing any of its own charge. This process is called charging by induction.



A metal sphere is supported on an insulation stand. When you bring a negatively charged rod near it, without actually touching it, the free electrons on the rod, and they shift toward the right, away from the rod. They cannot escape from the sphere because the supporting stand and the surrounding air are insulators. So we get excess negative charge at the right surface of the sphere and a deficiency of negative charge at the left surface. These excess charges are called induced charges.

Not all the free electrons move to the right surface of the sphere. As soon as any induced charge develops, it exerts forces toward the left on the other free electrons. These electrons are repelled by the negative induced charge on the right and attracted toward the positive induced charge on the left. The system reaches an equilibrium state in which the force toward the right on an electron, due to the charged rod, is just balanced by the force toward the left due to the induced charge. If we remove the charged rod, the free electrons shift back to the left, and the original neutral condition is restored.

1.11 Coulomb's Law

Charles Augustin de Coulomb (1736-1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction, as we discussed in Section 12.1. For point charges, charged bodies that are very small in comparison with the distance r between them, Coulomb found that the electric force is proportional to $1/r^2$. That is, when the distance r doubles, the force decreases 1/4 to of its initial value; when the distance is halved, the force increases to four times its initial value. The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by q or Q. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. Thus he could obtain one half, one quarter, and so on, of any initial charge. He found that the forces that two point charges q_1 and q_2 exert on each other are proportional to each charge and therefore are proportional to the product q_1q_2 of the two charges.

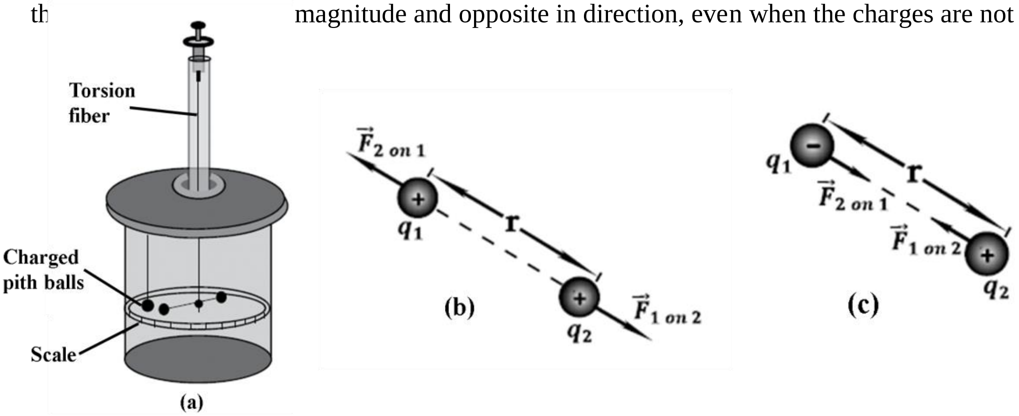
Thus Coulomb established what we now call Coulomb's law:

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

In mathematical terms, the magnitude F of the force that each of two point charges q_1 and q_2 a distance are apart exerts on the other can be expressed as

$$F = k\frac{|q_1 q_2|}{r} \qquad \text{... (i)}$$

Where k is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in because the charges can be either positive or negative, while the force magnitude F is always positive. The directions of the forces the two forces the two charges exert on each other are always along the line joining them. When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive. The two forces obey Newton's third law; th magnitude and opposite in direction, even when the charges are not equal.



The SI unit of electric charge is called one coulomb (1 C). In SI units the constant k in the last equation is

$$\mathrm{k = 8.987551787 \times 10^9 N.\frac{m^2}{C^2} \simeq 8.988 \times 10^9 N.m^2/C^2}$$

The value of k is known to such a large number of significant digits because this value is closely related to the speed of light in vacuum. As we discussed in Section 1.3, this speed is defined to be exactly $c = 2.99732458 \times 10^8\ m/s.$ The numerical value of k is defined in terms of c to be precisely

$$k = \left(10^{-7}\ N.\frac{s^2}{C^2}\right) c^2$$

In SI units we usually write the constant k in the last equation as $1/4\pi\epsilon_0$, where ϵ_0 ("epsilon-nought" or "epsilon zero") is another constant. This appears to complicate matters, but it actually simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \qquad \text{... (ii)}$$

(Coulomb's law : force between two point charges)

$$\epsilon_0 = 8.854 \times 10^{-12}\,\frac{c^2}{N}.m^2 \text{ and } \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^9\ N.m^2/C^2$$

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ N.m^2/C^2$$

One Coulomb represents the negative of the total charge of about 6×10^{18} electrons. For comparison, a copper cube 1 cm on a side contains about 2.4×10^{23} electrons. About 1019 electrons pass through the glowing filament of a flashlight bulb every second.

1.12 Superposition of Forces

Coulomb's law as we have stated it describes only the interaction of two point charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would exert individually. This important property, called the principle of superposition of forces, holds for any number of charges. By using this principle, we can apply Coulomb's law to any collection of charges. Several of the examples at the end of this section show applications of the superposition principle. Strictly speaking, Coulomb's law as we have stated it should be used only for point charges in a vacuum. If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material.

Illustration

Two points charges, q_1 = +25nC and q_2 = –75nC, are separated by a distance of 3.0 cm. Find the magnitude and direction of (a) the electric force that q_1 exerts on q_2; (b) the electric force that q_2 exerts on q_1.

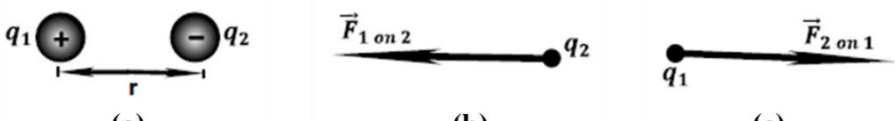
Solution

(a) Converting charge to coulombs and distance to meters, the magnitude of the force that q_1 exerts on q_2 is

$$F_{1\text{ on }2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$= \left(9.0 \times 10^9\ N.\frac{m^2}{C^2}\right)\frac{|(+25\times10^{-19}C)(-75\times10^{-9}C)|}{(0.030m)^2}$$

$$= 0.019\ \text{N}$$



(a) (b) (c)

(b) Remember that Newton‘s third law applies to the electric force. Even though the charges have different magnitudes, the magnitude of the force that q_2 exerts on q_1 is the same as the magnitude of the force that q_1 exerts on q_2:

$$F_{2\text{ on }1} = 0.019\ N$$

Newton‘s third law also states that the direction of the force that q_2 exerts on q_1 is exactly opposite the direction of the force that q_1 exerts on q_2.

Illustration

Illustration

Two identical conducting spheres of a small radius are charged and placed 25 cm apart. It is observed that they attract each other with a force 1.728N. The spheres are now connected by a thin conducting wire and the wire is then removed. It is observed that the spheres now repel each other with a force 0.576 N. Find the initial charges on the spheres?

Solution

Since the spheres attract initially, they carry charges of opposite nature, say q_1 and – q_2.

Magnitude of force, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

Here $\quad r = 25\ cm = 25 \times 10^{-2}m$

$\quad F = 1.725\ N$

$\therefore \quad 1.728 = 9 \times 10^9 \frac{q_1q_2}{(25\times10^{-2})^2}$

or $\quad q_1q_2 = 12 \times 10^{-12} \quad \ldots(1)$

When the spheres are connected by a conducting connecting wire, charges on them redistribute so as to attain a common electric potential. If q_1' and q_2' are the charges after redistribution, equality of electric potential requires

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{R}$$

$\left[\text{Potential on surface } V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}, \text{where R is the radius}\right]$

So $\quad q_1' = q_2'$

Since there is no loss of charge during redistribution, total charge on the system remains the same.

Here $\quad q_1' + q_2' = q_1 + (-q_2) = q_1 - q_2$

but $\quad q_1' = q_2'$

$\therefore \quad q_1' = q_2' = \frac{q_1 - q_2}{2}$

The charges are now observed to repel with a force 0.576 N.

$\therefore \quad \frac{1}{4\pi\epsilon_0}\frac{q_1' q_2'}{r^2} = 0.576$

or $\quad 9 \times 10^9 \frac{\left(\frac{q_1 - q_2}{2}\right)^2}{(25 \times 10^{-2})^2} = 0.576$

$\therefore \quad q_1 - q_2 = 4 \times 10^{-6} \qquad \ldots (2)$

Substituting q_2 from Eq. (2) in Eq. (1)

$q_1(q_1 - 4 \times 10^{-6}) = 12 \times 10^{-12}$

$q_1^2 - (4 \times 10^{-6})q_1 - 12 \times 10^{-12} = 0$

Solving, the roots are $6 \times 10^{-6}\ C$ and $-\ 2 \times 10^{-6}\ C$

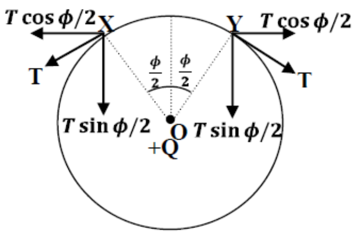
Hence, the two charges are $6 \times 10^{-6}\ C$ and $-\ 2 \times 10^{-6}\ C$.

Illustration

A thin ring of radius ‘r’ and made of a conducting material has +q distributed uniformly along its length. Find the increase in the tension of the ring if a point charge +Q is kept at the centre of ring?

Solution

Referring to figure, XY is a small element of length dl and subtends angle ϕ at the centre. Since charge on the ring is uniformly distributed,



$\therefore$ Linear charge density $= \frac{q}{2\pi r}$

and charge on small element XY,

$$dq = \frac{q}{2\pi r} dl \qquad \ldots (1)$$

Repulsive force applied by +Q (kept at centre) on this element X

$$dF = \frac{1}{4\pi\epsilon_0}\frac{Q\,dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{2\pi r}\frac{dl}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qq\,dl}{2\pi r^3}$$

Let T be the increase in tension so as to balance this outward (repulsive) force. The outward repulsion is balanced by $2T\sin\frac{\phi}{2}$,

$$2T\sin\frac{\phi}{2} = \frac{1}{4\pi\epsilon_0}\frac{Qq\,dl}{2\pi r^3}$$

Since dl is a short length element, angle subtended by dl at O, i.e. ϕ, is quite small. Hence $\sin\frac{\phi}{2} \approx \frac{\phi}{2}$.

$\therefore \quad 2T\frac{\phi}{2} = \frac{1}{4\pi\epsilon_0}\frac{Qq\,dl}{2\pi r^3}$

Since $\quad \phi = \frac{dl}{r}$

$\therefore \qquad 2T\frac{dl}{2r} = \frac{1}{4\pi\epsilon_0}\frac{Qq\,dl}{2\pi r^3}$

$\therefore \qquad T = Qq/8\pi^2\varepsilon_0 r^2$

Illustration

A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 g/mol. Let us now take two pieces of copper each weighting 10 g. Let us transfer one electron from one piece to another for every 1000 atoms in a piece. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?

$\left[e = 1.6 \times 10^{-19}\ C, \left(\frac{1}{4\pi\varepsilon_0}\right) = 9 \times 10^9 \text{ and Avogardro's number} = 6 \times \frac{10^{23}}{\text{mol}}\right]$

Solution

A 1 mole, i.e., 63.5 g of copper 6×10^{23} contains atoms, the number of atoms in 10 g copper will be

$$\frac{6\times10^{23}\times10}{63.5} = 9.45 \times 10^{22}$$

and as out of 1000 atoms 1 electron is transferred from one piece of copper to the other, the total electrons transferred from one piece to the other will be

$$\left(\frac{1}{1000}\right) \times 9.45 \times 10^{22} = 9.45 \times 10^{19}$$

Due to transfer of these electrons one piece will become negative while the other positive with magnitude of charge

$$q = ne = (9.45 \times 10^{19}) \times (1.6 \times 10^{-19}) = 15.12\ C$$

So treating each piece of copper as point charge, electric force between them from Coulomb's law when they are 10 cm apart will be,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = 9 \times 10^9\,\frac{15.12\times15.12}{(10\times10^{-2})^2}$$

$$= 2.06 \times 10^{14} N$$

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