Chapter 1

Kinematics

Day – 1

Distance and Displacement

Suppose and insect is at a point A (x_1, y_1, z_1) at $t = t_1$. It reaches at point $B(x_2, y_2, z_2)$ at $t = t_2$ through path ACB with respect to the frame shown in figure. The actual length of curved path ACB is the distance travelled by the insect in time $\Delta t = t_2 - t_1$.

If we connect point A (initial position) and point B (final position) by a straight line, then the length of straight line AB gives the magnitude of displacement of insect in time interval $\Delta t = t_2 - t_1$.

The direction of displacement is directed from A to B through the straight line AB. From the concept of vector, the position vector of $\vec{A}$ is $\vec{r}_A = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$ and that of B is $\vec{r}_B = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$.
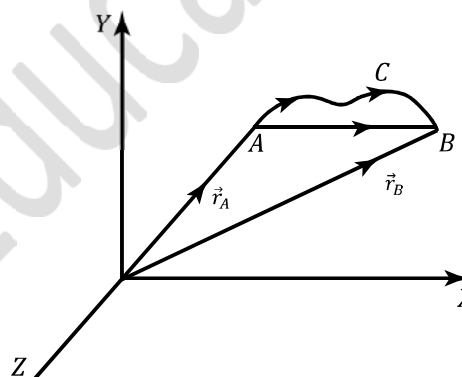
According to addition law of vectors,

$$\vec{r}_A + \overrightarrow{AB} = \vec{r}_B$$
$$\therefore \qquad \overrightarrow{AB} = \vec{r}_B - \vec{r}_A$$
$$\Rightarrow \qquad = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

The magnitude of displacement is

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Some Conceptual Points

(i) Distance is a scalar quantity.

(ii) Distance never be negative.

(iii) For moving body, distance is always greater than zero.

(iv) Distance never be equal to displacement.

(v) Displacement is a vector quantity.

(vi) If a body is moving continuously in a given direction on a straight line, then the magnitude of displacement is equal to distance.

(vii)Generally, the magnitude of displacement is less or equal to distance.

(viii)Many paths are possible between two points. For different paths between two points, distances are different but magnitudes of displacement are same.

(ix)The slope of distance-time graph is always grater or equal to zero.

(x)The slope of displacement-time graph may be negative.
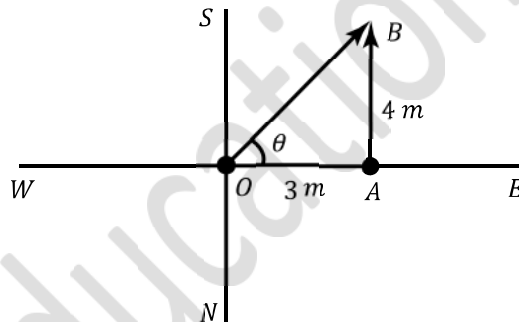
Illustration

A man walks 3 m in east direction, then 4 m in north direction. Find distance covered and the displacement covered by man.

Solution

The distance covered by man is the length of path =3m+4m=7m

Let the man starts from 0 and reaches finally at B (shown in figure). $O\vec{B}$ Represents the displacement of man. From figure,

$$\left|O\vec{B}\right| = (OA)^2 + (AB)^2$$

$$\Rightarrow (3m)^2 + (4m)^2 = 5m$$

And

$$\tan\theta = \frac{4m}{3m}$$

$$= \frac{4}{3}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

The displacement is directed at an angle $\tan^{-1} 4/3$ north of east.

Average speed and Average velocity

Suppose we wish to calculate the average speed and average velocity of the insect (in section2) between $t = t_1$ and $t = t_2$. From the path (shown in figure) we see that at $t = t_1$, the position of the insect is $A(x_1, y_1, z_1)$ and at $t = t_2$ the position of the insect is $B(x_2, y_2, z_2)$.

The average speed is defined as total distance travelled by a body in a particular time interval divided by the time interval. Thus, the average speed of the insect is

$$v_{av} = \frac{\text{the length of curve } ACB}{t_2 - t_1}$$

The average velocity is defined as total displacement travelled by a body is a particular time interval divided by the time interval.

Thus, the average velocity of the insect in the time interval $t_2 - t_1$ is

$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1}$$

$$= \frac{\overrightarrow{r_B} - \overrightarrow{r_A}}{t_2 - t_1}$$

$$= \frac{(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}}{t_2-t_1}$$

Some Important Points

(i) Velocity is a vector quantity while speed is a scalar quantity.

(ii) If a particle travels equal distances at speeds $v_1, v_2, v_3 \dots etc.$ respectively, then the average speed is harmonic mean of individual speeds.

(iii) If a particle moves a distance at speed v_1 and comes back with speed v_2 then $v_{av} = 2v_1v_2/v_1 + v_2$
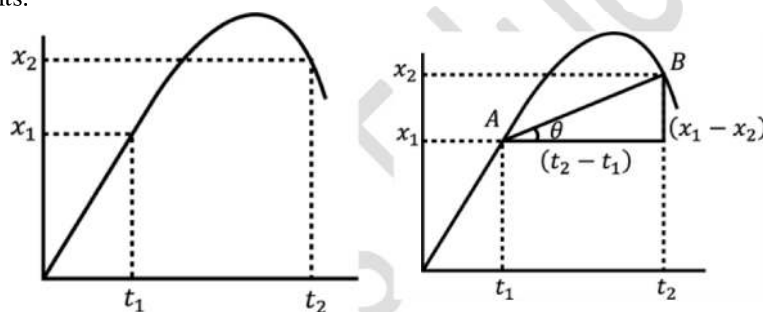
But

$$\vec{v}_{av} = 0$$

(iv) If a particle moves in two equal intervals of time at different speeds v_1 and v_2 respectively, then

$$v_{av} = \frac{v_1+v_2}{2}$$

(v) The average velocity between two points in a time interval can be obtained from a position versus time graph by calculating the slope of the straight line joining the co-ordinates of the two points.

The graph [shown in fig(a)],describes the motion of a particle moving along x-axis(along a straight line) suppose we wish to calculate the average velocity between $t = t_1$ and $t = t_2$ The slope of chord AB [shown in fig] gives the average velocity. Mathematically,

$$v_{av} = \tan\theta = \frac{x_2-x_1}{t_2-t_1}$$

(vi) The area of speed time graph gives distance.

(vii) The area of velocity time graph gives displacement

(viii) Speed can never be negative.

Instantaneous Velocity

Instantaneous velocity is defined as the average velocity over smaller and smaller interval of time. suppose position of a particle at t is $\vec{r}$ and at $t + \Delta t$ is $\vec{r} + \Delta\vec{r}$. the average velocity of the particle for time interval Δt is $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$ from our definition of instantaneous velocity, Δt should be smaller and smaller.

Thus,

Instantaneous velocity is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

If position of a particle at an instant t is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then

x- Component of velocity is

$$v_x = \frac{dx}{dt}$$

y- component of velocity is

$$y_x = \frac{dy}{dt}$$

Z- component of velocity is

$$v_z = \frac{dz}{dt}$$

Thus, velocity of the particle is

$$\vec{V} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$
$$= \frac{dx}{dt}\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \frac{dz}{dt}\hat{k}$$

Some Important Points

(i) Average velocity may or may not be equal to instantaneous velocity.

(ii) If body moves with constant velocity, the instantaneous velocity is equal to average velocity.

(iii) The instantaneous speed is equal to modulus of instantaneous velocity.

(iv) Distance travelled by particle is

$$s = \int |\vec{V}|dt$$

(v) X- component of displacement is

$$\Delta t = \int v_x dt$$

y- component of displacement is

$$\Delta y = \int v_y dt$$

z- component of displacement is

$$\Delta z = \int v_z dt$$

Thus, displacement of particle is

$$\Delta \vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

(vi) If particle moves on a straight line, (along x- axis),

$$\text{then } v = \frac{dx}{dt}$$

(vii) The area of velocity – time graph gives displacement.

(viii)The area of speed –time graph gives distance.

(ix) The slope of tangent at position- time graph at a particular instant gives instantaneous velocity at that instant.

Question Practice Online

Day – 2

Average Acceleration and Instantaneous Acceleration

In general, when a body is moving, its velocity is not always the same. A body whose velocity is increasing is said to be accelerated.

Average acceleration is defined as change in velocity divided by the time interval.

Let us consider the motion of a particle, suppose that the particle has velocity $\vec{v}_1$ at $t = t_1$ and at a later time $t = t_2$ it has velocity $\hat{v}_2$. Thus, the average acceleration during time interval $\Delta t = t_2 - t_1$ is

$$a_{av} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

If the time interval approaches to zero, average acceleration is known as instantaneous acceleration mathematically,

$$a = \lim_{\Delta t - 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Some Important Point

(i)Acceleration a vector quantity.

(ii)Its unit is m/s^2

(iii)The slope of velocity- time graph in a particular

(iv)The area of acceleration- time graph in a particular time interval gives change in velocity in that time interval.

Problem Solving Strategy

Motion on a straight line (one dimensional motion)

Uniform velocity	motion with constant acceleration	motion with variable acceleration
(i)$s = vt$	(i)$s = \left(\frac{u+v}{2}\right)t$	(i)if $a = f(t), a = \frac{dv}{dt}$
(ii)$a = 0$	(ii)$s = ut + \frac{1}{2}at^2$	(ii) if $a = f(s)$, $a = v\frac{dv}{ds}$
	(iii)$v^2 = u^2 + 2as$	(iii)if $a\,f(v), a = \frac{dv}{dt}$
	$(iv)v = u + at$	(iv)$v = \frac{ds}{dt}$
	$(v)nth = u + (2n-1)a/2$	(v)$s = \int v\,dt$
	(vi)for retardation,$'$ a$'$ will be negative.	(vi)$v = \int a\,dt$

Motion in two or Three Dimension

A body is free to move in space. In this case. The initial position of body is taken as origin.

Any convenient co- ordinate system is chosen. Let us suppose that an instant t, the body is at point $p(x, y, z)$.

The position vector of the body is $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

Thus, velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

In this way,

And acceleration along x-axis is

$$a_x = \frac{dv_x}{dt}$$

The velocity along y-axis is

$$v_y = \frac{dy}{dt}$$

And the acceleration along y-axis is

$$a_y = \frac{dv_y}{dt}$$

Similarly,

$$v_z = \frac{dz}{dt}$$

And

$$a_z = \frac{dv_z}{dt}$$

The acceleration of the body is

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}.$$

Discussion

(i) (a) if a_x is constant,

$$v_x = u_x + a_x t$$
$$\Rightarrow x = u_x t + \frac{1}{2}ax t^2$$
$$\Rightarrow v^2 x = u^2 x + 2a_x x$$

(b) If a_x is variable,

$$x = \int v_x dt$$
$$\Rightarrow \int dv_x = \int a_x dt$$

(ii) (a) if a_y is constant,

$$y = u_y t + \frac{1}{2}a_y t^2$$
$$\Rightarrow v_y = u_y + a_y t$$
$$\Rightarrow v_y^2 = u_y^2 + 2a_y y$$

(b) if a_y is variable,

$$y = \int v_y dt$$
$$\Rightarrow \int dv_y = \int a_y$$

(iii)   (a) if a_z is constant,

$$v_z = u_z + a_z t$$
$$\Rightarrow z = u_z t + \frac{1}{2} a_z t^2$$
$$\Rightarrow v_z^2 = u_z^2 + 2 a_z z$$

(b) if a_z is variable,

$$z = \int v_z \, dt$$
$$\Rightarrow dv_z = \int a_z \, dt$$

If the motion of the body takes place in x-y plane,
Then

$$a_z = 0 \,, v_z = 0 \,, u_z = 0$$

Motion Under Gravity

The most familiar example of motion with constant acceleration on a straight line is motion in a vertical direction near the surface of earth. If air resistance is neglected, the acceleration, of such type of particle is gravitational acceleration which is nearly constant for a height negligible with respect to the radius of earth. The magnitude of gravitational acceleration near surface of earth is

$$g = 9.8 m/s^2 = 32 ft/s^2$$

Discussion

Case I:- If particle is moving upwards
In this case applicable kinematics relations are

(i) $v = u - gt$

(ii) $h = ut - \frac{1}{2} gt^2$

(iii) $v^2 = u^2 - 2gh$

(iv)   Here h is the vertical height of the particle In the upward direction. For maximum height attained by projectile

$$h = h_{max}, \, v = 0$$

i.e. $(0)^2 = u^2 - 2gh_{max}$

$\therefore$ $h_{max} = \frac{u^2}{2g}$

Case II:- If particle is moving vertically downwards:
In this case,

(i) $v = u + gt$

(ii) $v^2 = u^2 + 2gh$

(iii) $h = ut + \frac{1}{2} gt^2$

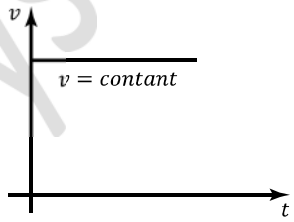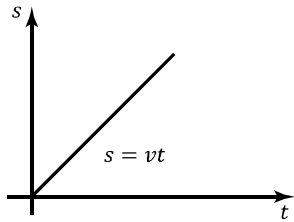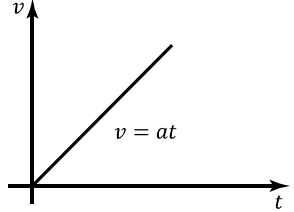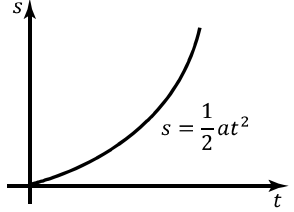Here, h is the vertical height of particle in downward direction

Day – 3

Graphs

The theory of graphs can be generalized and summarized in following six points

(i) A linear equation between x and y represents a straight line, e.g., $y = 4x - 2$, $y = 5x + 3$, $3x = y - 2$ equations represent straight line on $x - y$ graph.

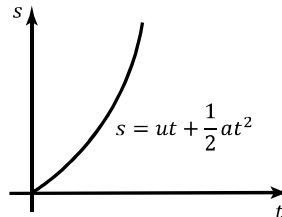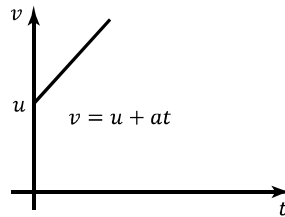(ii) $x \alpha y$ or $y = kx$ Represents a straight line passing through origin.

From the above six points we may conclude that in case of a one dimensional motion

(a) slope of displacement-time graph gives velocity $\left(as\ v = \frac{ds}{dt}\right)$.

(b) slope of velocity-time graph gives acceleration $\left(as\ a = \frac{dv}{dt}\right)$

(c) area under velocity-time graph gives displacement $(as\ ds = v\ dt)$

(d) area under acceleration-time graph gives change in velocity $(as\ dv = a\ dt)$.

(e) displacement-time graph in uniform motion is a straight line passing through origin, if displacement is zero at time $t = 0 (as\ s = vt)$.

(f) velocity-time graph is a straight line passing through origin in a uniformly accelerated motion if initial velocity $u = 0$ and a straight line not passing through origin if initial velocity $u \neq 0$ $(as\ v = u + at)$.

(g) displacement-time graph in uniformly accelerated of retarded motion is a parabola $\left(as\ s = ut \pm\right.$

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Now, we can plot $v - t$ and $s - t$ graphs of some standard results in tabular from as under. But note that all the following graphs are drawn for one dimensional motion with uniform velocity or with constant acceleration.

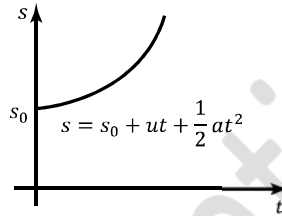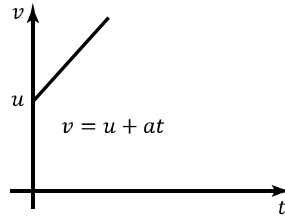| S. No.	Different cases	v-t Graph	s-t graph	Important points
1.	Uniform motion	$v = contant$	$s = vt$	(i) Slope of v-t graph $=v = constant$ (ii) In s-t graph $s = 0$ at $t = 0$
2.	Uniform accelerated motion with $u = 0$ and $s = 0$ at $t = 0$	$v = at$	$s = \frac{1}{2}at^2$	(i) $u = 0$, i.e., $v = 0$ at $t = 0$ (ii) a or slope of v-t graph is constant (iii) $u = 0$, i.e., slope of s-t graph at $t = 0$, should be zero

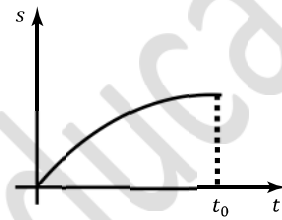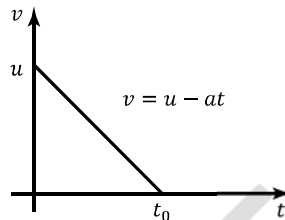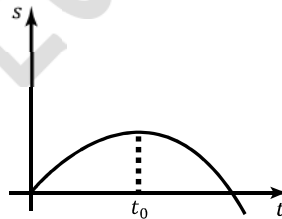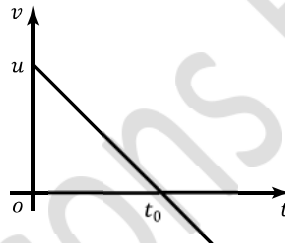|				
3.	Uniformly accelerated motion with $u \neq 0$ but $s = 0$	$v = u + at$	$s = ut + \frac{1}{2}at^2$	(i) $u \neq 0$, i.e., v or slope of v-t graph at $t = 0$, is not zero (ii) s or slope of s-t graph gradually goes on increasing *(iii)*
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$	$v = u + at$	$s = s_0 + ut + \frac{1}{2}at^2$	(i) $v = u$ at $t = 0$ *(ii)* $s = s_0$ at $t = 0$
5.	Uniformly retarded motion till velocity becomes zero	$v = u - at$		(i) Slope of s-t graph at $t = 0$ gives u (ii) Slope of s-t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero
6.	Uniformly retarded then accelerated in opposite direction.			(i) At time $t = t_0$, $v = 0$ or slope of s-t graph is zero (ii) In s-t graph slope or then increases with opposite sing.

Illustration

A rocket is fired vertically upwards with a net acceleration of 4 m/s² and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity-time and displacement-time graphs for the complete journey. Take g = 10 m/s².

Solution

In the graph,

$$v_A = at_{0A} = (4)(5) = 20 \, m/s$$
$$v_B = 0 = v_A - gt_{AB}$$
$$t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2s$$
$$t_{0AB} = (5+2)s = 7s$$

Now, s_{0AB} = area under v-t graph between 0 to 7 s

$$|s_{0AB}| = |s_{BC}| = \frac{1}{2}gt_{BC}^2$$
$$70 = \frac{1}{2}(10)t_{BC}^2$$
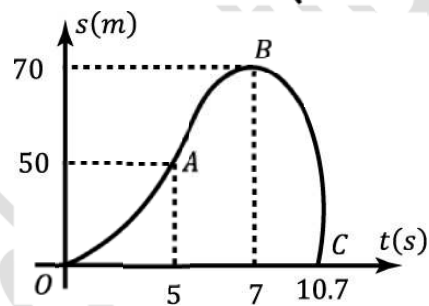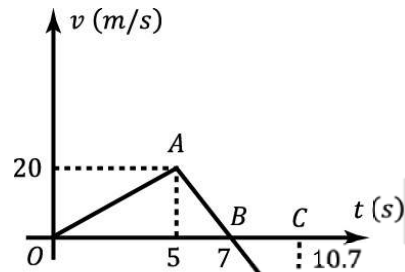$$t_{BC} = \sqrt{14} = 3.7s$$
$$t_{0BC} = 7 + 3.7 = 10.7s$$

Also

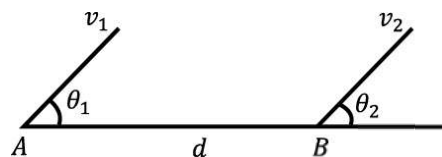$s_{OA} = area \; under \; v - t \; graph \; between \; OA$

$$= \frac{1}{2}(5)(20) = 50m$$

Question Practice Online

Velocity of Approach

if two particles A and B separated by a distance d at a certain instant of time move with velocities v_1 and v_2 at angles θ_1 and θ_2 with the direction $\vec{AB}$, the velocity by which the particle A approaches $B = v_1 \cos\theta_1 - v_2 \cos\theta_2$.
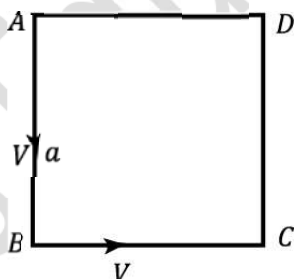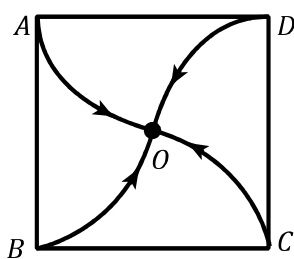
The angular velocity of B with respect to A

$$\Rightarrow \frac{v_2 \sin\theta_2 - v_1 \sin\theta_1}{d}$$

Illustration

Four particles are located at the corners of a square whose side equals a. They all start moving simultaneously with velocity v continually for the second, the second for the third, third for the fourth and fourth for the first. How soon will the particles converge

Solution

The paths of particles are shown in figure

The velocity of approach of A to B

$$\Rightarrow v - v \cos 90^0$$
$$\Rightarrow v - 0 = v$$

Day – 5

Projectile Motion

A familiar example of two dimensional motions is projective motion. If a stone is thrown from ground obliquely, it moves under the force of gravity (in the absence of air resistance) near the surface of earth. Such type of motion is known as projectile motion. We refer to such object as projectile. To analyses this type of motion, we will start with its acceleration. The motion of stone is under gravitational acceleration which is constant in magnitude as well as in direction.
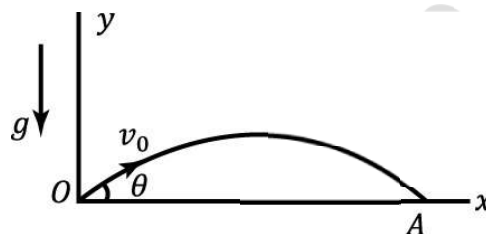
Now let us consider a projectile launched so that its initial velocity v_0 makes an angle θ with the horizontal (shown in figure). For discussion of motion, we take origin at the point of projection.

Horizontal direction as x-axis and vertical direction as $y - $ axis is taken.

The component of gravitational acceleration along $x - $ axis is

$$u_x = v_0 \cos\theta.$$

The component of gravitational acceleration along $y - $ axis is
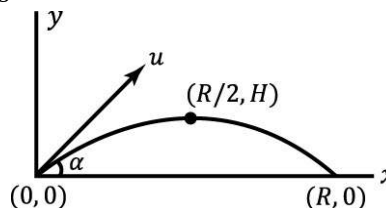
$$a_x = g\cos 90^0 = 0.$$

The component of initial velocity along $y - axis$ is

$$u_y - u_0 \sin\theta.$$

The acceleration along $y - axis$ is

$$a_y = -g$$

A particle is projected with a velocity u at an angle α to the horizontal, there being no force except gravity, which remains constant through its motion

$$\vec{u} = u\cos\alpha\hat{i} + u\sin\alpha\hat{j}$$
$$\Rightarrow \vec{a} = -g\hat{j}$$
$$\Rightarrow \vec{s} = x\hat{i} + y\hat{j}$$
$$\Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}\,t^2$$
$$x\hat{i} + y\hat{j} = (u\cos\alpha\hat{i} + u\sin\alpha\hat{j})t - \frac{1}{2}gt^2\hat{j}$$
$$\Rightarrow x = ut\cos\alpha, 0$$

$$\Rightarrow y = ut \sin\alpha - \frac{1}{2}gt^2$$

For the maximum height,

$$t = \frac{T}{2},$$

$$T = \frac{u^2 \sin \alpha}{g}, x = \frac{R}{2}, y = H$$

$$\therefore \qquad H = \frac{u^2 \sin^2 \alpha}{2g}$$

(a) For the range,

$$x = R, y = 0, t = T$$

$$\therefore \qquad R = \frac{u^2 \sin 2\alpha}{g}$$

(b):- (i):- If for two angles of projection α_1 and α_2, the speeds are same then ranges will be same. The condition is $\alpha_1 + \alpha_2 = 90^0$

(ii):- It particles be projected form the same point in the same plane so as to describe equal parabolas, the vertices of their paths lie on a parabola.

(iii):- The locus of the foci of all parabolas described by the particles projected simultaneously from the same point with equal velocity but in different directions is a circle

(iv):- The velocity acquired by a particle at any point of its path is the same as acquired by a particle in falling freely from the directory to that point.

(v):- A projectile will have maximum range when it is projected at an angle of 45^0 to the horizontal and the maximum range will be u^2/g

At the maximum range, $H = R_{max}/4$

(vi):- In the case of projectile motion, a the highest point, potential energy is maximum and is equal to $\frac{1}{2}mu^2 \sin^2 \alpha$.

(vii):- If the body is projected at an angle of 45^0 to the horizontal, at the highest point half of its mechanical energy is potential energy and rest is kinetic energy.

(viii):- The weight of a body in projectile motion is zero as it is freely falling body.

(ix):- If two projectiles A and B are projected under gravity, then the path of projectile A with respect to the projectile B is a straight line

Illustration

Show that there are two values of time for which a projectile is at the same height. Also show mathematically that the sum of these two times is equal to the time of flight.

Solution

For vertically upward motion of projectile,

$$y = (u \sin \alpha)t - \frac{1}{2}gt^2$$

Or

$$\Rightarrow \frac{1}{2}gt^2 - (u\sin \alpha)t + y = 0$$

This is a quadratic equation in t. Its roots are

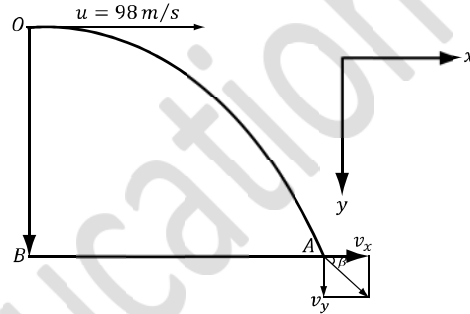$$t_1 = \frac{u \sin \alpha - \sqrt{u^2 \sin^2 \alpha - 2gy}}{g}$$

And

$$t_2 = \frac{u \sin\alpha + \sqrt{u^2 \sin^2 \alpha - 2gy}}{g}$$

$$\therefore t_1 + t_2 = \frac{2u \sin\alpha}{g} = T \quad \text{(time of flight of the projectile)}$$

Illustration

A projectile is fired horizontally with a velocity of $98 \ m/s$ from the top of a hill 490 m high. Find

(a) The time taken by the projectile to reach the ground

(b) The distance of the point where the particle hits the ground from foot of the hill and

(c) The velocity with which the projectile hits the ground. $g = 9.8 \ m/s^2$

Solution

In this problem we cannot apply the formulae of R, H and T directly. We will have to follow the three steps discussed in the

Theory.

Here, if will be more convenient to choose x and y directions as shown in figure.

Here, $u_x = 9.8 \ m/s$, $a_x = 0$, $u_y = 0 \ and \ a_y = g$

(a)　At

$$A, s_y = 490m. \text{ so applying}$$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$\therefore \quad 490 = 0 + \frac{1}{2}(9.8)t^2$$

$$\therefore \quad t = 10_s$$

(b)　　　　　$$BA = s_x$$

$$\Rightarrow u_x t + \frac{1}{2} a_x t^2$$

Or　　　　　　$$BA = (98)(10) + 0$$

Or　　　　　　$$BA = 980m$$

(c)　　　　　$$v_x = u_x = 9.8 \ m/s$$

$$v_y = u_y + a_y t$$

$$\Rightarrow 0 + (9.8)(10)$$

$$\Rightarrow 98 \ m/s$$

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$$\therefore v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \sqrt{(98)^2 + (98)^2}$$

$$\Rightarrow 98\sqrt{2}\,m/s$$

And

$$\tan\beta = \frac{v_y}{v_x}$$

$$\Rightarrow \frac{98}{98} = 1$$

$$\therefore \beta = 45^0$$

Thus the projectile hits the ground with a velocity $98\sqrt{2}\,m/s$ at an angle of $\beta = 45^0$ with horizontal as shown in fig.

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