Chapter 2 Three Dimensional Geometry

Day – 1

Three Dimensional Co-ordinate System

Position vector of a point on space

Let $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors (called base vectors) along OX, OY and OZ respectively.Let P(x,y,z) be a point in space, let the position of P be $\vec{r}$.

Then,

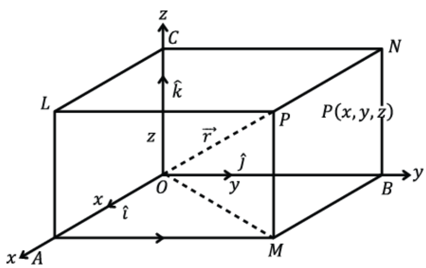
$$\vec{r} = \overrightarrow{OP}$$

$$\Rightarrow \quad \overrightarrow{OM} + \overrightarrow{MP}$$

$$\Rightarrow \quad \left(\overrightarrow{OA} + \overrightarrow{AM}\right) + \overrightarrow{MP}$$

$$\Rightarrow \quad \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$



Thus the position vector of a point P is;

$$x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Signs of Co-ordinates of a Point in Various Octants

Octant /Co-ordinate	OXYZ	OX'YZ	OXY'Z	OXYZ'	OX'Y'Z	OX'YZ'	OXY'Z'	OX'Y'Z'
X	+	-	+	+	-	-	+	-
Y	+	+	-	+	-	+	-	-
Z	+	+	+	-	+	-	-	-

Illustration

Planes are drawn parallel to the co-ordinate planes through the points (1,2,3) and (3,-4,-5). Find the lengths of the edges of the parallelepiped so formed.

Solution

Let P=(1,2,3) **,**Q=(3,-4,-5) through which planes are drawn parallel to the co-ordinate planes shown as,

∴PE=distance between parallel planes

ABCP and FQDE, i.e.(along z-axis)

$\Rightarrow |-5-3|$

$\Rightarrow 8$

PA= distance between parallel planes ABQF and PCDE

$\Rightarrow |3-2|$

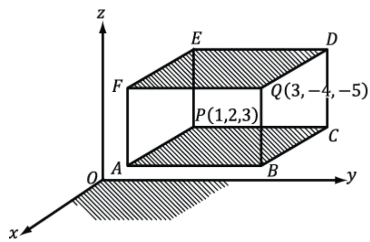
$\Rightarrow 1$

PC= distance between parallel planes BCDQ and APEF

$\Rightarrow |-4-2|$

$\Rightarrow 6$

∴lengths of edges of the parallelepiped are; 2,6,8



Distance Formula

The distance between the points $P(x_1, y_1, z_1)$ *and* $Q(x_2, y_2, z_2)$ is given by

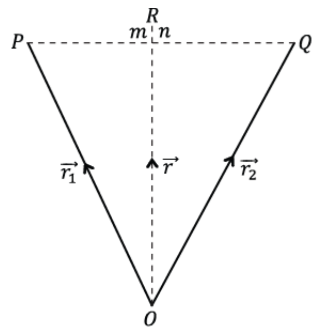
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Section Formula

(i):- **Section formula for internal division**

If R(x,y,z) is a point dividing the join of P(x_1,y_1,z_1) and Q(x_2,y_2,z_2) in the ratio m:n(**Internal Div.)** Then,

$$x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}, z = \frac{mz_2+nz_1}{m+n}$$



(ii):- **Section formula for external division**

The co-ordinates of a point R which divides the join of P(x_1,y_1,z_1) and Q(x_2,y_2,z_2) externally in the ratio

m:n are *;*

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_2}{m-n}\right)$$

(iii):- **Mid – point formula**

The co-ordinates of the mid-point of the join of P (x_1,y_1,z_1) and Q(x_2,y_2,z_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Illustration

Find the ratio in which 2x+3y+5z=1 divides the line joining the points (1,0,-3) and (1,-5,7)

Solution

Here, 2x+3y+5z =1 divides(1,0,-3) and (1,-5,7) in the ratio of k:1 at point P.

Then,

$$P = \left(\frac{k+1}{k+1}, \frac{-5k}{k+1}, \frac{7k-3}{k+1}\right)$$

Which must satisfy 2x+3y+5z=1

$$\Rightarrow 2\left(\frac{k+1}{k+1}\right) + 3\left(\frac{-5k}{k+1}\right) + 5\left(\frac{7k-3}{k+1}\right) = 1$$

$$\Rightarrow 2k + 2 - 15k + 35k - 15 = k + 1$$

$$\Rightarrow 21k = 14$$

$$\Rightarrow k = 2/3$$

∴2x+3y+5z=1 divides (1,0,-3) and (1,-5,7) in the ratio of 2:3.

Illustration

Find the locus of a point the sum of whose distance from(1,0,0) and (-1,0,0) is equal to 10.

Solution

Let the points A(1,0,0), B(-1,0,0) and P(x,y,z)

Given: PA+PB=10

$$\sqrt{(x-1)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+1)^2 + (y-o)^2 + (z-0)^2} = 10$$

$$\Rightarrow \sqrt{(x-1)^2 + y^2 + z^2}$$

$$\Rightarrow 10 - \sqrt{(x+1)^2 + y^2 + z^2}$$

Squaring both sides we get

$$\Rightarrow (x-1)^2 + y^2 + z^2$$

$$\Rightarrow 100 + (x+1)^2 + y^2 + z^2 - 20\sqrt{(x+1)^2 + y^2 + z^2}$$

$$\Rightarrow -4x - 100 = -20\sqrt{(x+1)^2 + y^2 + z^2}$$

$$\Rightarrow x + 25 = 5\sqrt{(x+1)^2 + y^2 + z^2}$$

Again squaring both sides we get

$$\Rightarrow x^2 + 50x + 625$$

$$\Rightarrow 25\{(x^2 + 2x + 1) + y^2 + z^2\}$$

$$\Rightarrow 24x^2 + 25y^2 + 25z^2 - 600 = 0$$

i.e. required equation of locus.

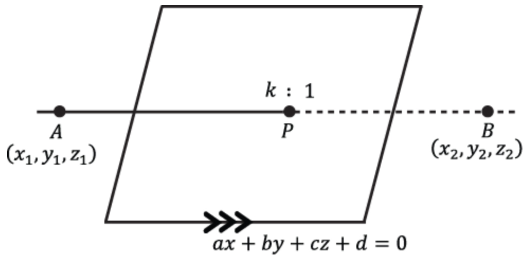
Illustration

Show that the plane ax+by+cz+d=0 divides the line joining (x_1,y_1,z_1) and (x_2,y_2,z_2) in the ratio of

$$\left(-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}\right)$$

Solution

Let the plane ax+by+cz+d=0 divides the line joining (x_1,y_1,z_1) and (x_2,y_2,z_2) in the ratio of k:1 as shown in figure;



$$\therefore co-ordinates\ of\ P\left(\frac{kx_2+x_1}{k+1},\frac{ky_2+y_1}{k+1},\frac{kz_2+z_1}{k+1}\right)\ must\ satisfy\ ax+by+cz+d=0$$

$$a\left(\frac{kx_2+x_1}{k+1}\right)+b\left(\frac{ky_2+y_1}{k+1}\right)+c\left(\frac{kz_2+z_1}{k+1}\right)+d=0$$

$$\Rightarrow a(kx_2+x_1)+b(ky_2+y_1)+c(kz_2+z_1)+d(k+1)=0$$

$$\Rightarrow k(ax_2+by_2+cz_2+d)+(ax_1+by_1+cz_1+d)=0$$

$$k=-\frac{(ax_1+by_1+cz_1+d)}{(ax_2+by_2+cz_2+d)}.$$

Direction Cosines and Direction Ratio's of a Vector

1. Direction cosines

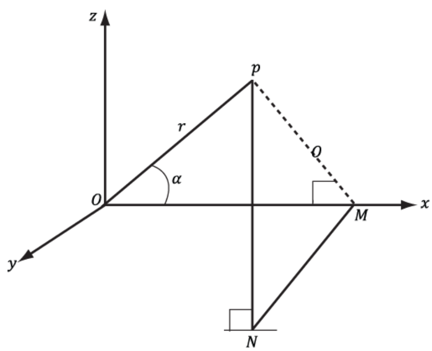
Ifα,β,γ are the angles which a vector $\overrightarrow{OP}$ makes with the positive directions of the co-ordinate axes OX,OY,OZ respectively, then $cos\,\alpha$, $cos\,\beta$, $cos\,\gamma$ are known as direction cosines of $\overrightarrow{OP}$ and are generally denoted by letters l,m,n respectively.

Thus,$l=cos\,\alpha$,$m=cos\,\beta$,$n=cos\,\gamma$. The angles α,β,γ are known as direction angles and they satisfy the condition $0\le\alpha,\beta,\gamma\le\pi$

Direction cosines of x-axis are (1,0,0)

Direction cosines of y-axis are (0,1,0)

Direction cosines of z-axis are (0,0,1)



Co-ordinates of P are ($r\,cos\,\alpha$, $r\,cos\,\beta$, $r\,cos\,\gamma$)

$$x=r\,cos\,\alpha=lr$$

$$y=rcos\,\beta=mr$$

$$z=rcos\,\gamma=nr$$

(a) If l, m, n are direction cosines of a vector, then $l^2 + m^2 + n^2 = 1$

(b) $\vec{r} = |\vec{r}|(l\hat{\imath} + m\hat{\jmath} + n\hat{k})$ and $\hat{r} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$

2. Direction Ratios

Let l, m, n be the direction cosines of a vector $\vec{r}$ and a, b, c be three numbers such that a, b, c are proportional to l, m, n

i.e,

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} k \ or \ (l, m, n) = (ka, kb, kc)$$

$$\Rightarrow (a, b, c) are \ direction \ ratios.$$

"That shows there can be infinitely many direction ratios for a given vector, but the direction cosines are unique"

Let a, b, c be direction ratios of a vector $\vec{r}$ having direction cosines l, m, n

Then,

$$l = \lambda a, m = \lambda b, n = \lambda c \ (by \ definition\ )$$

$$= \therefore l^2 + m^2 + n^2 = 1$$

$$\Rightarrow a^2\lambda^2 + b^2\lambda^2 + c^2\lambda^2 = 1$$

$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

So,

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

(a) If $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ be a vector having direction cosines l, m, n Then

$$l = \frac{a}{|\vec{r}|}, m = \frac{b}{|\vec{r}|}, n = \frac{c}{|\vec{r}|}$$

(b) Direction ratios of the line joining two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $x_2 - x_1, y_2 - y - 1, z_2 - z_1$.

3. Directions cosines of parallel vectors

Let $\vec{a}$ and $\vec{b}$ two parallel vectors, Then $\vec{b} = \lambda\vec{a}$ for some λ.

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, then\ \vec{b} = \lambda\vec{a}$

$$\Rightarrow \vec{b} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$$

This show that $\vec{b}$ has direction ratios $\lambda a_1, \lambda a_2, \lambda a_3$

I,e.,

$$a_1, a_2, a_3 \ becous \ \lambda a_1 : \lambda a_2 : \lambda a_3 = a_1 : a_2 : a_3$$

Thus $\vec{a}$ and $\vec{b}$ have equal direction ratios and hence equal direction cosines also.

4. If direction ration's of $\vec{r}$ are a, b, c

$$\vec{r} = \frac{|\vec{r}|}{\sqrt{a^2+b^2+c^2}}\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$$

5. Projection of $\vec{r}$ on the co-ordinate axes are

$$l|\vec{r}|, m|\vec{r}|, n|\vec{r}|$$

6. The projection of segment joining the points $p(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on a line with direction cosines l, m, n

is

$$(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$$

7. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two concurrent lines, then the direction cosines of the lines bisecting the angles between them are proportional to $l_1 \pm l_2, m_1 \pm m_2, n_1 \pm n_2$

Angle between two vectors in terms of direction cosines and direction ratios

Let $\vec{a}$ and $\vec{b}$ two given vectors with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 respectively. Then,

$$\hat{a} = l_1\hat{\imath} + m_1\hat{\jmath} + n_1\hat{k} \ and \ \hat{b} = l_2\hat{\imath} + m_2\hat{\jmath} + n_2\hat{k}$$

$$cos\,\theta \ = \frac{\hat{a}.\hat{b}}{|\hat{a}||\hat{b}|}$$

$$cos\,\theta = \frac{l_1l_2 + m_1m_2 + n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}$$

$$\Rightarrow cos\,\theta = \ l_1l_2 + m_1m_2 + n_1n_2$$

1. Acute angle θ between the two lines having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 is given by

$$cos\,\theta = |l_1l_2 + m_1m_2 + n_1n_2|$$

2. If a_1, b_1, c_1 and a_2, b_2, c_2 be the direction ratios of two lines then the acute angle θ between them is given by

$$cos\,\theta \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2\sqrt{a_2^2+b_2^2+c_2^2}}}$$

3. Two lines having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are

(a) Perpendicular if and only if $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(b) Parallel if and only if $l_1/l_2 = m_1/m_2 = n_1/n_2$

4. Two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2

(a) Perpendicular if and only if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(b) Parallel if and only if $a_1/a_2 = b_1/b_2 = c_1/c_2$

Illustration

What are the direction cosines of line which is equally inclined to axes.

Solution

If α, β, γ are the angles that a line makes with the co-ordinate axe, then if they are equally inclined

$$\Rightarrow \alpha = \beta = \gamma$$

Also

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow cos^2\alpha + cos^2\beta + cos^2\gamma = 1$$

Or

$$cos^2\alpha + cos^2\alpha + cos^2\alpha = 1$$

Or

$$3\,cos^2\alpha = 1$$

$$\Rightarrow cos\,\alpha = \pm\frac{1}{\sqrt{3}} = cos\,\beta = cos\,\gamma$$

$\therefore direction\ cosines\ are$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) or \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

Illustration

A line OP through origin O is inclined at 30^0 and 45^0 to OX and OY respectively. Find the angle at which is inclined to OZ.

Solution

Let l, m, n be the direction cosine of the given vector.

$$l^2 + m^2 + n^2 = 1$$

Where

$$\alpha = 30^0, \beta = 45^0$$

$$\therefore cos^2\alpha + cos^2\beta + cos^2\gamma = 1$$

$$\Rightarrow cos^2 30^0 + cos^2 45^0 + cos^2\gamma = 1$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + cos^2\gamma = 1$$

$$\Rightarrow cos^2\gamma = 1 - \frac{3}{4} - \frac{1}{2}$$

$$\Rightarrow cos^2\gamma = \frac{4-3-2}{4}$$

$$\Rightarrow cos^2\gamma = -\frac{1}{4} which\ is\ not\ possible.$$

$\therefore$ There exists no point which is inclined to 30^0 to x- axis and 45^0 to y-axis

Illustration

A vector $\vec{r}$ has length 21 and direction ratios 2,-3,6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an obtuse angle with x-axis.

Solution

Here, direction ratio's are 2,-3,6

$\therefore$Direction cosines can be written as

$$\left\{\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda\right\}$$

$$2\lambda, -3\lambda, 6\lambda$$

Where

$$(2\lambda)^2 + (-3\lambda)^2 + (6\lambda)^2 = 1$$

As

$$49\lambda^2 = 1\ (\therefore l^2 + m^2 += 1)$$

$$\Rightarrow \lambda = \pm\frac{1}{7}$$

$\therefore$ Direction cosines are

$$\left(\pm\frac{2}{7}, \pm\frac{3}{7}, \pm\frac{6}{7}\right)$$

But it makes obtuse angle with x-axis $\Rightarrow l<0$.

$\therefore$ Direction cosines are

$$\left(-\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)$$

Also

$$\vec{r} = |\vec{r}|\left(l\hat{\imath} + m\hat{\jmath} + n\hat{k}\right)$$

$$\Rightarrow \vec{r} = 21\left(-\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}\right)\ given\ |\vec{r}| = 21$$

$$\vec{r} = 3\left(-2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right)$$

So the component of $\vec{r}$ along x, y and z-axis are $-6\hat{\imath}, +9\hat{\jmath}$ and $-18\hat{k}$ respectively.

Illustration

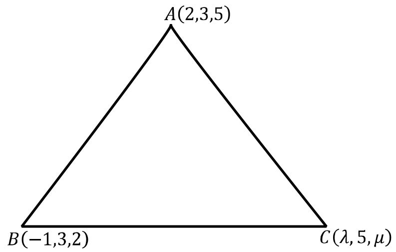
ABC is a triangle where A=(2,3,5) B=(-1,3,2) and C= $(\lambda, 5, \mu)$. If the median through A is equally inclined to the axis then find the values of λ and μ. Also, find the area of the ΔABC.

Solution

Here, A(2,3,5)B(-1,3,2) C($\lambda, 5, \mu$)

$\therefore$ mid-point of

BC=$(\lambda - 1/2\,, 4, 2 + \mu/2)$



Dr's of median through A

$$\left(\frac{\lambda-1}{2} - 2, 4-3, \frac{2+\mu}{2} - 5\right)$$

i.e.,

$$\left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right)$$

As the median through A is equally inclined to the axis.

$\therefore$ dr's will be constant and equals to k.

$$\frac{\frac{\lambda-5}{2}}{k} = \frac{1}{k} = \frac{\frac{\mu-8}{2}}{k} \Rightarrow \frac{\lambda-5}{2} = 1 = 1\frac{\mu-8}{2}$$

$$\Rightarrow \lambda = 7, \mu = 10$$

$$\therefore C = (7,5,10)$$

Now, area of ΔABC= $1/2\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$= \frac{1}{2}\left|\left(-3\hat{\imath} - 3\hat{k}\right) \times (5\hat{\imath} + 2\hat{\jmath})\right| = \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 0 & -3 \\ 5 & 2 & 5 \end{vmatrix}$$

$$= \frac{1}{2}\left|6\hat{\imath} - 6\hat{k}\right| = 3\sqrt{2}$$

Illustration

If Q be the foot of perpendicular from P(2,4,3)on the line joining the points A(1,2,4) and B(3,4,5) find the co-ordinates of Q

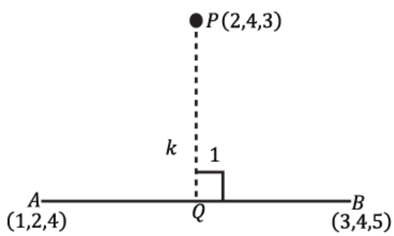
Solution

Let Q divides AB internally in the ratio k:1

$$\Rightarrow Q = \left(\frac{3k+1}{k+1}, \frac{4k+2}{k+1}, \frac{5k+4}{k+1}\right)$$

$\therefore$ Direction ratios of PQ are

$$\Rightarrow \frac{3k+1}{k+1} - 2, \frac{4k+2}{k+1} - 4, \frac{5k+4}{k+1} - 3$$



Or

$\Rightarrow \frac{k-1}{k+1}, \frac{-2}{k+1}, \frac{2k+1}{k+1}$

And direction ratios of AB are (3-1, 4-2, 5-4) or (2, 2, 1)

Since

$PQ \perp AB$

$$\Rightarrow 2\left(\frac{k-1}{k+1}\right) + 2\left(\frac{-2}{k+1}\right) + 1\left(\frac{2k+1}{k+1}\right) = 0$$
$$\Rightarrow 2k - 2 - 4 + 2k + 1 = 0$$
$$\Rightarrow 4k - 5 = 0$$
$$\Rightarrow k = \frac{5}{4}$$

$\therefore$ $$Q = \left(\frac{3.\frac{5}{4}+1}{\frac{5}{4}+1}, \frac{4.\frac{5}{4}+2}{\frac{5}{4}+1}, \frac{5.\frac{5}{4}+4}{\frac{5}{4}+1}\right)' Q = \left(\frac{19}{9}, \frac{28}{9}, \frac{41}{9}\right)$$

$\therefore$ foot of perpendicular

$$\left(\frac{19}{9}, \frac{28}{9}, \frac{41}{9}\right)$$

Illustration

If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two mutually perpendicular lines, shows that the direction cosines of the line perpendicular to both of them are $m_2n_2 - m_2n_1; n_1l_2 - n_2l_1; l_1m_2 - l_2m_1$

Solution

Let l, m, n be the direction cosines of the line perpendicular to both the given lines

$$\therefore ll_1 + mm_1 + nn_1 = 0$$
$$ll_2 + mm_2 + nn_2 = 0$$

Solving by cross-multiplication method

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1} = k$$
$$\therefore l = k(m_1n_2 - m_2n_1); m = k(n_1l_2 - n_2l_1); n = k(l_1m_2 - l_2m_1)$$

Squaring and adding; we have

$$l^2 + m^2 + n^2 = k^2\{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2\} + (l_1m_2 - l_2m_1^2)$$
$$\Rightarrow 1 = k^2\{sin^2\theta\}$$

Where θ is the angle between the given lines as we know

$$sin\,\theta\sqrt{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2}$$

Where $a_1, b_1, c_1; a_2, b_2, c_2$ are direction cosines.

$$1 = k^2.1\ (as\ \theta = 90^0, given\ )$$
$$\Rightarrow k = 1$$

Hence, direction cosines of a line perpendicular to both of them are

$$m_1n_2 - m_2n_1, n_1l_2 - n_2l_1; l_1m_2 - l_2m_1$$

Illustration

If $l_1, m_1, n_1; l_2, m_2, n_2$ be the d.c.'s of two concurrent lines, show that the d.c's of the line bisecting the angles between them are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$

Solution

Let, direction cosines of $OA = (l_1, m_1, n_1)$ and direction cosines of $OB = (l_2, m_2, n_2)$ Taking two points on l and m such that $OA = OB = r$

Let C be the mid-point of AB

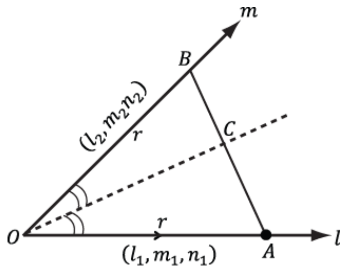
Then OC is the bisector of the angle AOB

Now, co-ordinates of A are (l_1r, m_1r, n_1r) coordinates of B are (l_2r, m_2r, n_2r)

$\therefore$ The co-ordinates of C are

$$\frac{(l_1+l_2)}{2}, \frac{(m_1+m_2)}{2}, \frac{(n_1+n_2)}{2}$$

Hence, the direction cosines of OC are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$.



Illustration

Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to (1,-2,-2) and (0,2,1).

Solution

If l, m, n be the direction cosines of the line perpendicular to the given line, then

$$l.(1) + m(-2) + n(-2) = 0$$

$$\Rightarrow l - 2m - 2n = 0 \ldots (i)$$

And

$$l.0 + m.2 + n.1 = 0$$

$$0 + 2m + n = 0 \ldots (ii)$$

Then form (i) and (ii) by cross multiplication, we get

$$\frac{l}{2} = \frac{m}{-1} = \frac{n}{2}$$

$$\Rightarrow \frac{l}{2} = \frac{m}{-1} = \frac{n}{2} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{4+1+4}} = \frac{1}{3} (\because l^2 + m^2 + n^2 = 1)$$

$$\Rightarrow l = \frac{2}{3}, m = -\frac{1}{3}, n = \frac{2}{3}$$

Practice Question Online