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# Chapter 7 Hyperbola

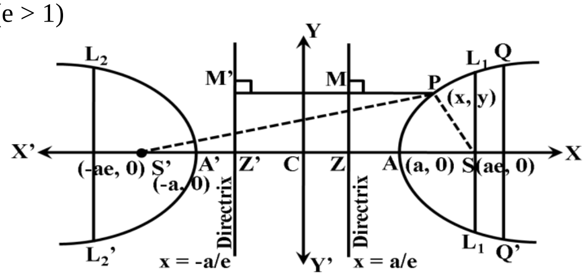
**Day – 1**

## 1. Hyperbola

**Definition**

The locus of a point which moves in a plane such that its distance from a fixed point (i.e., focus) is e times its distance from a fixed line (i.e., directrix) is known as ***Hyperbola***.

For Hyperbola (e > 1)




### 1.1 Standard Equation of Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$b^2 = a^2(e^2 - 1)$$

### 1.2 General Equation

The equation of Hyperbola whose focus is ($h$, $k$) and directrix is $lx + my + n = 0$ and whose eccentricity e is

$$(x - h)^2 + (y - k)^2 = e^2 \frac{(lx+my+n)^2}{l^2+m^2}$$

### 1.3 The Foci and Directrix of a Hyperbola

Foci (± ae, 0)   Directrix $= \pm\frac{a}{e}$,

**Note :** Distance between foci SS" = 2ae

Distance between directrixes = $\frac{2a}{e}$,

### 1.4 Some Terms Related to Hyperbola

The equation of the Hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

**1.** Centre C (0, 0) ⇒ All chord passing through C are bisected at C.

**2.** Eccentricity $b^2 = a^2(e^2 - 1)$ for Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

**3.** Foci: = (± ae, 0)

**4.** Axes: The point A(a, 0) and A'(–a, 0) are called vertices of the Hyperbola and line AA' is called transverse axis and the line perpendicular to transverse axis and passing through C(0, 0) is called the conjugate axis.

**5.** Double ordinate: Any line perpendicular to the transverse axis is called Double ordinate.

If the abscissa of Q is *h* then ordinates of Q are

$$\frac{y^2}{b^2} = \frac{h^2}{a^2} - 1 \Rightarrow y = \pm\frac{b}{a}\sqrt{h^2 - a^2}$$

$y = \frac{b}{a}\sqrt{h^2 - a^2}$ (I$^{st}$ quadrent) $y = -\frac{b}{a}\sqrt{h^2 - a^2}$ (IV quadrent)

Hence the coordinate of double ordinate are $\left(h, \frac{b}{a}\sqrt{h^2 - a^2}\right)$ and $\left(h, -\frac{b}{a}\sqrt{h^2 - a^2}\right)$.

**6.** Latus Rectum: The double ordinates LL' and $L_1L_1$' are the latus rectum of the Hyperbola. These lines are perpendicular to transverse axis AA' and through the foci S and S' respectively.

## 1.5 Length of Latus Rectum

Let *LL'* = 2*k*    *LS* = *L'S* = *k*

Coordinate of L, L' an (*ae*, *k*) (*ae*, –*k*)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

$$k^2 = b^2(e^2 - 1) = b^2\left(\frac{b^2}{a^2}\right)$$

$$k = \frac{b^2}{a}$$

So length $LL' = 2k = \frac{2b^2}{a}$,

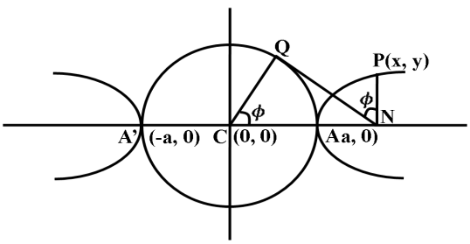
$\Rightarrow$ End points of Latus Rectum $\left(ae, \frac{b^2}{a}\right)\left(ae, -\frac{b^2}{a}\right)\left(-ae, \frac{b^2}{a}\right)\left(-ae, -\frac{b^2}{a}\right)$.

**7.** Focal Chord: A chord of the Hyperbola passing through its focus is called a focal chord.

**8.** Parametric Equation of the Hyperbola

Equation of Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Equation of auxiliary circle $x^2 + y^2 = a^2$




## Proof

Let P(*x*, *y*) be any point are the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ draw PN perpendicular to x axis.

Let NQ be a tangent to the auxiliary circle $x^2 + y^2 = a^2$ join CQ.

Here P and Q are the corresponding points of the Hyperbola and the auxiliary circle φ is the eccentric angle of P (u ≤ θ ≤ 2π).

$$Q(a\cos\theta, a\sin\theta)$$

$$x = CN = \frac{CN}{CQ} \times CQ = \sec\varphi, a$$

$$x = a\sec\varphi$$

Put this on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ so

$$\frac{a^2 \sec^2 \varphi}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2 \tan^2 \varphi$$

$$y = \pm b \tan \varphi$$

P lie in first quadrant so $(a \sec \phi, b \tan \phi)$.

**Note:-** Equation of chord joining point $P(a \sec \varphi_1, b \tan \varphi_1) Q(a \sec \varphi_2, b \tan \varphi_2)$

$$\frac{x}{a} \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) - \frac{y}{b} \sin\left(\frac{\varphi_1 + \varphi_2}{2}\right) = \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right)$$

If it is a focal chord when it passes through ($ae$, 0) or (–$ae$, 0)

$$e \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) = \cos\left(\frac{\varphi_1 + \varphi_2}{2}\right)$$

$$\frac{\cos\left(\frac{\varphi_1 - \varphi_2}{2}\right)}{\cos\left(\frac{\varphi_1 + \varphi_2}{2}\right)} = \frac{1}{e}$$

$$\frac{\cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) - \cos\left(\frac{\varphi_1 + \varphi_2}{2}\right)}{\cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) + \cos\left(\frac{\varphi_1 + \varphi_2}{2}\right)} = \frac{1-e}{1+e}$$

$$\tan\frac{\varphi_1}{2}, \tan\frac{\varphi_2}{2} = \frac{1-e}{1+e}$$

If it passes through (–$ae$, 0) then

$$\tan\frac{\varphi_1}{2}, \tan\frac{\varphi_2}{2} = \frac{1+e}{1-e}$$

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