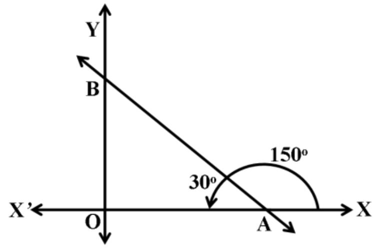
Chapter 2 The Straight Line

Day – 1

1. Angle of inclination of line

The angle of inclination of a line is the measure of the angle between the x – axis and the line measured in the anticlockwise direction.



Note:-

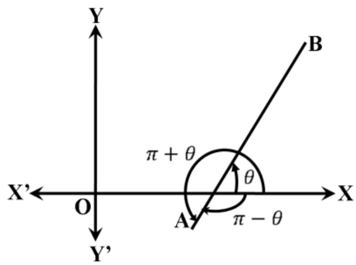
(i) When two lines are parallel, they have the same inclination.

(ii) The inclination of a line which is parallel to x – axis or coinciding with x – axis is 0^o.

(iii) The angle of inclination of the line lies between 0^o and 180^o i.e., $0 < \theta \leq \pi$ and $\theta \neq \frac{\pi}{2}$.

1.1 Slope of gradient of a line

If inclination of a line is ($\theta \neq 90^o$) then tan θ is called the slope or gradient of the line. It is usually denoted by m. θ is positive or negative according as it is measured in anticlockwise or clockwise direction.



Note:-

(1) Slope of a line is not the angle but is the tangent of the inclination of the line.

(2) If a line is parallel to x –axis then its slope = tan 0^o = 0.

(3) Slope of a line parallel to y- axis or perpendicular to x- axis is not defined. Whenever we say that slope of a line is not defined.

(4) If a line is equally inclined to the axes, then it will make an angle of 45^o or 135^o with the positive direction of x – axis. Slope in this case will be tan 45^o or tan 135^o. i.e.,±1.

Theorem

If P(x_1, y_1) and Q(x_2, y_2) are two points on a line l, then the slope m of the line l is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1},\ x_1 \neq x_2$$

If $x_1 = x_2$, then m is not defined. In that case the line is perpendicular to x – axis.

Illustration

Find the inclination of the whose slope is $-\frac{1}{\sqrt{3}}$.

Solution

Let α be the Inclination of a line then its slope = tan α

∴ $\tan\alpha = \frac{1}{\sqrt{3}} = -\tan 30^o = \tan(180^o - 30^o) = \tan 150^o$

⇒ $\alpha = 150^o$.

Illustration

Find the slope of the line through the points (4, – 6), (–2, – 4).

Solution

Slope of the line $\mathrm{m} = \frac{-5-(-6)}{-2-(4)} = -\frac{1}{6}$.

Illustration

Determine λ, so that 2 is the slope of the line through (2, 5) and (λ, 3).

Solution

Slope of the line joining (2, 5) and (λ, 3)

$$= \frac{3-5}{\lambda-2} = -\frac{2}{\lambda-2} = 2 \text{ (given)}$$

⇒ $-2 = 2\lambda - 4$

⇒ $2\lambda = 2$

∴ $\lambda = 1$

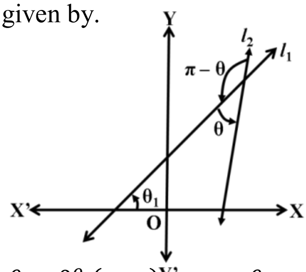
1.2 Angle between two lines

Theorem

The acute angle θ between the lines having slopes m1 and m2 is given by.

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$



Corollary

(1) If two lines, whose slopes are m1 and m2 are parallel, iff $\theta = 0^o$ (or π) $\Leftrightarrow \tan\theta \Leftrightarrow m_1 = m_2$.

Thus when two lines are parallel, their slopes are equal.

(2) If two lines, whose slopes are m1 and m2 are perpendicular, iff

$\theta = \frac{\pi}{2}\left(or -\frac{\pi}{2}\right) \Leftrightarrow \cot\theta = 0 \Leftrightarrow m_1 m_2 = -1.$

Thus, when two lines are perpendicular, the product of their slopes is -1 .The slope of each is the negative reciprocal of the slope of other i.e., if m is the slope of a line, then slope of a line perpendicular to it is $-\frac{1}{m}$.

Illustration

The angle between two lines is $\frac{\pi}{4}$ and the slope of one them is $\frac{1}{2}$. Find the slope of the other line.

Solution

If θ be the acute angle between the lines with slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Let $\theta = \frac{\pi}{4}$ and $m_1 = \frac{1}{2}$

Then $\tan\frac{\pi}{4} = \left|\frac{\frac{1}{2} - m_2}{1 + \frac{1}{2} m_2}\right|$

$\Rightarrow \quad 1 = \left|\frac{1 - 2m_2}{2 + m_2}\right|$

$\Rightarrow \quad \frac{1 - 2m_2}{2 + m_2} = \pm 1$

Taking positive sign then, $1 - 2m_2 = 2 + m_2$

$\therefore \quad m_2 = -\frac{1}{3}$

And taking negative sign then,

$1 - 2m_2 = 2 - m_2$

$\therefore \quad m_2 = 3$

Hence the slope of the other line is either $-\frac{1}{3}$ or 3.

1.3 Line parallel to co – ordination axes

(i) Equation of a line parallel to y – axis :-

Let l be a straight line parallel to y – axis and at a distance a from it, a being the directed distance of the line from the y – axis. Therefore, the line lies on the right of y – axis if a > 0 and if a < 0 then the line would lies on the left of y – axis.

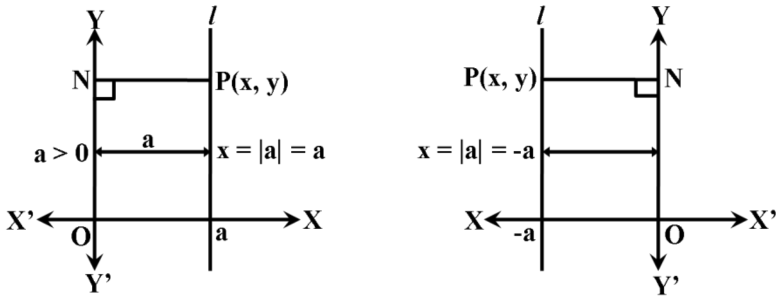
Here, $|a| = a$

Let P(x, y) be any point on the line l, then

$x = a$ is the required equation.

Remark

In particular equation of y –axis is x = 0 (∵ a = 0)

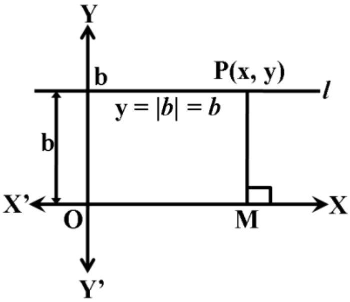


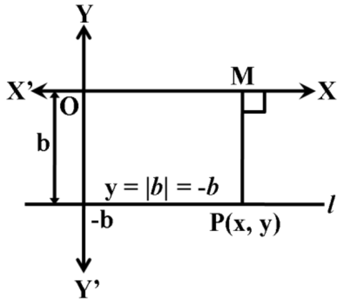
Note:- A line to y–axis, at a distance from it and on the negative side of y–axis, then its equation is x = – a.

(ii) Equation of a line parallel to x – axis:-

Let l be a straight line parallel to x – axis and at a distance b from it, b being the direct distance of the line from the x – axis. Therefore, the line lies above the x – axis, if b > 0 and if b < 0, then the line would lie below the x – axis.

Let P(x, y) be any point on the line l, then y = b is the required equation.





Remark

In particular equation of x–axis is y–axis (∵ a = 0)

Note:- A line parallel to x–axis at a distance b from it and is on the negative side of x–axis, then its equation is y = – b.

Illustration

Find the equation of the straight line parallel to y–axis and at a distance (i)3 units to the right (ii) 2 units to the left.

Solution

(i) Equation of straight line parallel to y – axis at a distance a units to the right is x = a.

∴ Required equation is x = 3

(ii) Equation of straight line parallel to y –axis at a distance a units to the left is x = – a.

∴ Required equation is x = –2.

Illustration

Find the equation of the straight line parallel to x–axis and at a distance (i) 5 units above the x–axis (ii) 9 units below the x–axis.

Solution

(i) Equation of straight line parallel to x – axis at a distance b units to the right is x = a.

∴ Required equation is x = 3

(ii) Equation of straight line parallel to x –axis at a distance b units below the x – axis is y = – b.

∴ Required equation is x = –2.

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