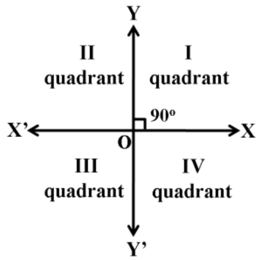
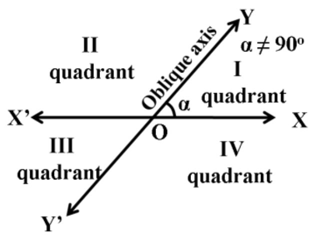
Chapter 1 Coordinate Systems and Coordinates

Day – 1

1. Co- ordinate Axes





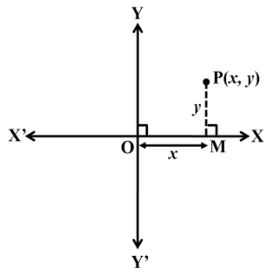
1.1 Rectangular Cartesian co – ordinates of a point

(i) The ordinate of every point on x – axis is 0.

(ii) The abscissa of every point on y – axis is 0.

(iii) The abscissa and ordinate of the origin O(0, 0) are both zero.

(iv) The abscissa and ordinate of a point are at perpendicular distance from y – axis and x – axis respectively.



1.2 Distance Between Two Points

Theorem

$$|P| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$$

The distance PQ between the points P(x_1, y_1) and Q(x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Corollary – 1

The above formula is true for all positions of the points (i.e., either points are not in the 1st quadrant) keeping in mind, the proper signs of their co – ordinates.

Corollary – 2

The distance of the point P(x_1, y_1) and O (0, 0) is given by

$$|P| = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{(x^2 + y^2)}$$

Corollary –3

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Corollary –4

(i) If PQ is parallel to x – axis, then $y_1 = y_2$ and so

$$|P| = \sqrt{(x_1 - x_2)^2} = |x_2 - x_1|$$

(i) If PQ is parallel to y – axis, then $x_1 = x_2$ and so

$$|P| = \sqrt{(y_1 - y_2)^2} = |y_2 - y_1|$$

Note:- (1) When three points are given and it is required to:

(i) An isosceles triangle, show that two of its sides (or two angles) are equal.

(ii) An equilateral triangle, show that its all sides are equal or each angle is of 60^0.

(iii) A right angle triangle, show that the sum of the squares of two sides is equal to square of the side.

(iv) An isosceles right angled triangle, show that two of its sides are equal and the sum of the squares of two equal to the square of the third side.

(v) A scalene triangle, show that its all sides are unequal.

(2) When four points are given and it required to:

(i) A square, show that the four sides are equal and the diagonals are also equal.

(ii) A rhombus, show that the opposite are equal and the diagonals are also equal.

(iii) A rectangle, show that the opposite sides are equal the diagonals are also equal.

(iv) A parallelogram, show that the opposite sides are equal and the diagonals are not equal.

Illustration

Prove that the distance of the point $(a\cos\alpha, a\sin\alpha)$ from the origin is independent of α.

Solution

Let $P \equiv (a\cos\alpha, a\sin\alpha)$ and $O \equiv (0, 0)$

Then $|P| \equiv \sqrt{(a\cos\alpha - 0)^2 + (a\sin\alpha - 0)^2}$

$$= \sqrt{(a\cos^2\alpha + a^2\sin^2\alpha)} = \sqrt{a^2(\cos^2\alpha + \sin^2\alpha)} = \sqrt{a^2}$$

$= |a|$, which is independent of α

Illustration

If the point (x, y) be equidistant from the points (6, – 1) and (2, 3) prove that x – y = 3.

Solution

Let $\mathrm{P} \equiv (\mathrm{x}, \mathrm{y}), \mathrm{A} \equiv (6, -1)$ and $\mathrm{B} \equiv (2, 3)$

By the given conditions, PA = PB

$\Rightarrow \quad \sqrt{(x-6)^2 + (y+1)^2} = \sqrt{(x-2)^2 + (y-3)^2}$

or $\quad (x-6)^2 + (y+1)^2 = (x-2)^2 + (y-3)^2$

or $\quad x^2 - 12x + 36 + y^2 + 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$

or $\quad 8x - 8y = 24$ or $x - y = 3$.

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